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LOCAL PROPERTIES OF SOLUTIONS OF ELLIPTIC EQUATIONS DEPENDING ON LOCAL PROPERTIES OF THE DATA*

LUCIO BOCCARDO† AND TOMMASO LEONORI‡

Neil: "... ma misi me per l'alto mare aperto sol con un legno";
qui trovasti "quella compagna picciola da la qual non fui diserto."
(Dante: Inferno XXVI)

Abstract. In this paper we deal with local properties of solutions of the boundary value problem

$$\begin{cases} -\mathrm{div}(a(x,u,\nabla u)) = \mu & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

where the left hand side is a Leray-Lions operator and μ a Radon measure. In particular we look at properties of the solution away from the set where the datum is singular.

Key words. Nonlinear elliptic equations, local estimates, measure data.

AMS subject classifications. 35J65 (35J60, 35J25)

1. Introduction and main results. This paper deals with properties of solutions of nonlinear boundary value problems of the type

$$\begin{cases} -\mathrm{div}(a(x,u,\nabla u)) = \mu & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{1.1}$$

or

$$\begin{cases} -\mathrm{div}(a(x,u,\nabla u)) = f(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{1.2}$$

where Ω is a bounded, open subset of $\mathbb{R}^N$, $N > 2$, the right hand side is either a bounded Radon measure μ or a summable function f and the partial differential operator A is defined as

$$A(v) = -\mathrm{div}\,(a(x,v,\nabla v))$$

where $a : \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}^N$ is a Carathéodory function (that is, measurable with respect to x in Ω for every (s,ξ) in $\mathbb{R} \times \mathbb{R}^N$, and continuous with respect to (s,ξ) in $\mathbb{R} \times \mathbb{R}^N$ for almost every x in Ω). We assume that there exist two real positive constants α and β, such that for almost every x in Ω, for every s in $\mathbb{R}$, for every ξ and ξ' in $\mathbb{R}^N$ $(\xi \neq \xi')$,

$$a(x,s,\xi)\cdot\xi \geq \alpha|\xi|^2, \tag{1.3}$$

$$|a(x,s,\xi)| \leq \beta|\xi|, \tag{1.4}$$

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†Dipartimento di Matematica, Università di Roma 1, Piazza A. Moro 2, 00185 Roma, Italia (boccardo@mat.uniroma1.it).
‡CMUC, Departamento de Matemática, Universidade de Coimbra, 3001-454 Coimbra, Portugal (leonori@mat.uc.pt).