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LARGE EIGENVALUES AND TRACES OF STURM-LIOUVILLE EQUATIONS ON STAR-SHAPED GRAPHS*

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Abstract. In this paper, we consider the spectral problem of small vibrations of a graph consisting of d, $d \geq 2, d \in \mathbf{N}$, joint inhomogeneous smooth strings which can be reduced to the Sturm-Liouville boundary value problem on a graph. This problem occurs also in quantum mechanics. For the Sturm-Liouville problem on the compact metric graph consisting of d segments of equal length with the Dirichlet or Neumann boundary conditions at the pendant vertices and Kirchhoff condition at the central vertex, we first derive the asymptotic expressions of its large eigenvalues and obtain precise descriptions for the formulae of the square root of the large eigenvalues up to the $O(1/n)$-term. In addition, regularized trace formulae of operators are established with residue techniques in complex analysis.

Key words. Compact metric graph, Sturm-Liouville problem, Kirchhoff condition, eigenvalue asymptotics, trace formula

AMS subject classifications. 34B24, 34L20, 47E05

1. Setting of the problem.

A Quantum graph is given by a differential (self-adjoint) operator on a metric graph, i.e., the domain of the operator is a function space, each element in the space satisfying certain boundary conditions at the vertices. Differential operator on a metric graph(quantum graph) is a rather new and rapidly-developing area of modern mathematical physics. Such operators can be used to describe the motion of quantum particles confined to certain low dimensional structures. Spectral and scattering properties of Schrödinger operator in such structures have attracted considerable attention during past years. Many relevant models of nano-structure have been put out, see [27, 28, 30, 31, 32, 33]. An extensive survey of physical systems, giving rise to boundary value problems on graphs, can be found in the bibliography. Second order boundary value problems on finite graphs arise naturally from quantum mechanics and circuit theory [4, 20].

Recently, the spectral problems of quantum graphs have become a rapidly-developing field of mathematics and mathematical physics, and spectral properties of quantum graphs and different inverse problems have been studied in both forward [9, 14, 30, 31, 32, 33, 34, 44, 46, 50], and inverse [3, 7, 11, 35, 45, 48, 49, 53, 55, 56], etc. Some results on trace formulae and inverse scattering problems for Laplacians on metric graphs have appeared [6, 20, 26, 29, 36, 42, 51, 54], etc.

In this paper, we consider the following boundary value problems for the Sturm-Liouville equations on star-shaped metric graphs (i.e., a tree domain with exactly one central vertex) consisting of d segments of equal length:

$$-y_j'' + q_j(x)y_j = \lambda y_j, \quad j = 1, 2, \cdots, d; \ d \geq 2, d \in \mathbf{N}, \tag{1.1}$$

which are subject to the boundary conditions

$$y_j(0) = 0, \ j = 1, 2, \cdots, d \tag{1.2}$$

or

$$y_j'(0) = 0, \ j = 1, 2, \cdots, d, \tag{1.3}$$

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