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OSCILLATION CRITERIA FOR SECOND-ORDER NONLINEAR SELF-ADJOINT DIFFERENTIAL EQUATIONS*

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Dedicated to Professor Joel Smoller on the occasion of his 70th birthday

Abstract. Our concern is to solve the oscillation problem for the nonlinear self-adjoint equation $(a(t)x')' + b(t)g(x) = 0$, where $g(x)$ satisfies the Signum condition $xg(x) > 0$ if $x \neq 0$, but is not imposed such monotonicity as superlinear or sublinear. The problem has not been solved for the critical cases:

$$\liminf_{|x|\to 0} \frac{g(x)}{x} < \frac{1}{4} < \limsup_{|x|\to 0} \frac{g(x)}{x}, \quad and \quad \liminf_{|x|\to\infty} \frac{g(x)}{x} < \frac{1}{4} < \limsup_{|x|\to\infty} \frac{g(x)}{x},$$

which are more difficult, by now. We concentrate our attention on this point and give some answers. Sufficient conditions are given for all nontrivial solutions to be oscillatory.

Key words. Oscillation, nonlinear self-adjoint differential equations, Liénard-type system.

AMS subject classifications. Primary 34C10, 34C15; Secondary 34A12

1. Introduction.

In this paper, we are concerned to obtain some oscillation criteria for the nonlinear self-adjoint differential equation

$$(a(t)x')' + b(t)g(x) = 0, \tag{1.1}$$

where $a(t)$ and $b(t)$ are positive, continuous, and locally of bounded variation on some half-line $[\alpha, \infty)$, and $g(x)$ is continuous on $\mathbf{R}$ and

$$xg(x) > 0, \quad for \quad x \neq 0. \tag{1.2}$$

We assume that uniqueness is guaranteed for the solutions of (1.1) to the initial value problem. In [21, Appendix] the authors have proved that all solutions of (1.1) are continuable in the future time. Hence, it is worth while to discuss whether solutions of (1.1) are oscillatory or not.

A nontrivial solution $x(t)$ of (1.1) is said to be *oscillatory* if there exists a sequence t_k tending to infinity such that $x(t_k) = 0$. Otherwise, the solution is said to be *nonoscillatory.* For brevity, Eq.(1.1) is called *oscillatory* (respectively *nonoscillatory*) in case all nontrivial solutions are oscillatory(respectively nonoscillatory).
Equation (1.1) naturally includes the nonlinear equation

$$x'' + c(t)g(x) = 0, \quad t > 0, \tag{1.3}$$

as a special case.

Over the past few decades, a grate deal of efforts has been made on the oscillation and nonoscillation of solutions of (1.1)(or (1.3)). Those results can be found in [1-25] and the references cited therein. For example, there are many studies on the oscillation for the Emden-Fowler differential equation

$$x'' + a(t)|x|^{\gamma} sgnx = 0, \tag{1.4}$$

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