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LEVEL SET DYNAMICS AND THE NON-BLOWUP OF THE 2D QUASI-GEOSTROPHIC EQUATION*

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Abstract. In this article we apply the technique proposed in Deng-Hou-Yu [7] to study the level set dynamics of the 2D quasi-geostrophic equation. Under certain assumptions on the local geometric regularity of the level sets of θ, we obtain global regularity results with improved growth estimate on $\left|\nabla^{\perp}\theta\right|$. We further perform numerical simulations to study the local geometric properties of the level sets near the region of maximum $\left|\nabla^{\perp}\theta\right|$. The numerical results indicate that the assumptions on the local geometric regularity of the level sets of θ in our theorems are satisfied. Therefore these theorems provide a good explanation of the double exponential growth of $\left|\nabla^{\perp}\theta\right|$ observed in this and past numerical simulations.

Key words. Quasi-geostrophic equation, finite time blow-up, geometric properties, global existence

AMS subject classifications. Primary 76B03; Secondary 35L60, 35M10

1. Introduction.

The study of global existence/finite-time blow-up of the two-dimensional quasi-geostrophic (subsequently referred to as 2D QG for simplicity) equation has been an active research area in the past ten years, partly due to its close connection to the 3D incompressible Euler equations (Constantin-Majda-Tabak [2], Cordoba [5], Cordoba-Fefferman [6]). The 2D QG equation has its origin in modeling rotating fluids on the earth surface (Pedlosky [10]). The equation describes the transportation of a scalar quantity θ:

$$D_t\theta \equiv \theta_t + u \cdot \nabla\theta = 0 \tag{1}$$

with initial conditions $\theta\mid_{t=0}=\theta_0$. The relation between θ and the velocity u is given by

$$u = \nabla^{\perp}\psi, \quad \psi = (-\triangle)^{-\frac{1}{2}}(-\theta) \tag{2}$$

where

$$\nabla^{\perp}\psi \equiv \left(-\frac{\partial\psi}{\partial x_2}, \frac{\partial\psi}{\partial x_1}\right)^T \tag{3}$$

and

$$(-\triangle)^{-\frac{1}{2}}\psi \equiv \int e^{2\pi i x\cdot k}\frac{1}{2\pi|k|}\hat{\psi}(k)\,\mathrm{d}k \tag{4}$$

where $\hat{\psi}(k) = \int e^{-2\pi i x\cdot k}\psi(x)\,\mathrm{d}x$ is the Fourier transform of $\psi(x)$.

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