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WELL-POSEDNESS OF THE IDEAL MHD SYSTEM IN CRITICAL BESOV SPACES*

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Abstract. In this paper we study the ideal incompressible magneto-hydrodynamics system, and prove the local existence and uniqueness of solutions in critical Besov spaces $B^{1+n/p}_{p,1}$ for $1 \leq p \leq \infty$.

Key words. Magneto-hydrodynamics system, critical Besov space, existence and uniqueness

AMS subject classifications. 76W05, 74H20, 74H25

1. Introduction. We are concerned with the following ideal magneto-hydrodynamics (MHD) system for the homogeneous incompressible fluid flows and magnetic fields

$$u_t + (u \cdot \nabla)u - (b \cdot \nabla)b - \nabla\pi = 0 \tag{1.1}$$

$$b_t + (u \cdot \nabla)b - (b \cdot \nabla)u = 0 \tag{1.2}$$

$$\text{div} u = 0, \quad \text{div} b = 0 \tag{1.3}$$

with initial data

$$u(x,0) = u_0(x), \tag{1.4}$$

$$b(x,0) = b_0(x). \tag{1.5}$$

Here $u(x,t) = (u_1(x,t), u_2(x,t), \cdots, u_n(x,t))$ is the velocity of the fluid flows , $b(x,t) = (b_1(x,t), b_2(x,t), \cdots, b_n(x,t))$ is the magnetic field, and $\pi(x,t) = p(x,t) + \frac{1}{2}|b(x,t)|^2$ is the total pressure for $x \in \mathbb{R}^n$, $t \geq 0$ and $u_0(x)$ and $b_0(x)$ are the initial velocity and initial magnetic field satisfying divu_0=0, divb_0=0, respectively. For simplicity, we have set the Reynolds number, magnetic Reynolds number and the corresponding coefficients to be constant 1 by scaling transformation.

In the case of Euler equations, the existence and uniqueness of solutions to Euler equations have been studied by many authors (see J.-Y. Chemin [3] and reference there). Recently, Vishik [11], H.C. Park and Y.J. Park [6] obtained the existence and uniqueness of solutions of the incompressible Euler equations in critical Besov spaces. Vishik considered Euler equations in space dimension 2 and proved the global well-posedness in critical Besov space $B^{1+2/p}_{p,1}$, $1 < p < \infty$ by transport equation and the invariance of vorticity. For the ideal magneto-hydrodynamics system, the method Vishik used is not valid, and it is more complicated because of the couple effect between velocity $u(x,t)$ and magneto fields $b(x,t)$. The existence of the classical solution for MHD system was shown by Kozono [4] in the bounded domain, See also [9]. In BMO^{-1} and bmo^{-1} spaces, Miao, Yuan and Zhang proved the global existence and uniqueness of solution to the incompressible MHD system for small initial data [5]. In the case of Sobolev spaces $W^{k,p}$, the existence and uniqueness results for

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