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NONEXISTENCE OF POSITIVE SOLUTIONS FOR SOME FULLY NONLINEAR ELLIPTIC EQUATIONS*

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Dedicated to Joel Smoller on his 70th birthday

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It is well known that

$$\Delta u \geq u^p \quad \text{in } \mathbb{R}^n \tag{1}$$

has no positive solution if $p > 1$. For a proof, see for example Osserman [9], Loewner and Nirenberg [7] and Brezis [2]. We extend this result to some fully nonlinear elliptic equations. Some related problems will also be studied.

Let us fix some notations. For each $1 \leq k \leq n$ let

$$\sigma_k(\lambda) = \sum_{1 \leq i_1 < \cdots < i_k \leq n} \lambda_{i_1} \cdots \lambda_{i_k}, \qquad \lambda = (\lambda_1, \cdots, \lambda_n) \in \mathbb{R}^n,$$

denote the kth elementary symmetric function, and let Γ_k denote the connected component of $\{\lambda \in \mathbb{R}^n : \sigma_k(\lambda) > 0\}$ containing the positive cone $\{\lambda \in \mathbb{R}^n : \lambda_1 > 0, \cdots, \lambda_n > 0\}$. It is well known that $\Gamma_k = \{\lambda \in \mathbb{R}^n : \sigma_l(\lambda) > 0, 1 \leq l \leq k\}$. Let $S^{n\times n}$ denote the set of $n \times n$ real symmetric matrices. For any $A \in S^{n\times n}$ we denote by $\lambda(A)$ the eigenvalues of A.

Throughout this note we will assume that $\Gamma \subset \mathbb{R}^n$ is an open convex symmetric cone with vertex at the origin satisfying $\Gamma_n \subset \Gamma \subset \Gamma_1$. Moreover, we also assume that f is a continuous function defined on $\overline{\Gamma}$ verifying the following properties:

$$f \text{ is homogeneous of degree one on } \Gamma, \tag{2}$$

$$f \text{ is symmetric in } \lambda = (\lambda_1, \cdots, \lambda_n) \in \Gamma, \tag{3}$$

and

$$f \text{ is monotonically increasing in each variable on } \Gamma. \tag{4}$$

Given a smooth positive function u defined in $\mathbb{R}^n$ with $n \geq 3$, we may introduce

$$A^u = -\frac{2}{n-2} u^{-\frac{n+2}{n-2}} D^2 u + \frac{2n}{(n-2)^2} u^{-\frac{2n}{n-2}} Du \otimes Du - \frac{2}{(n-2)^2} u^{-\frac{2n}{n-2}} |Du|^2 I, \tag{5}$$

where I is the $n \times n$ identity matrix, and Du and D^2u denote the gradient and the Hessian of u respectively. This operator appears in the recent work on conformally invariant elliptic equations and the σ_k-Yamabe problems in conformal geometry, see for example [4, 11].

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