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GLOBAL WEAK SOLUTIONS TO 1D COMPRESSIBLE ISENTROPIC NAVIER-STOKES EQUATIONS WITH DENSITY-DEPENDENT VISCOSITY*

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1. Introduction.

We establish the global existence and uniqueness of weak solutions to the Navier-Stokes equations for a one-dimensional isentropic viscous gas with a jump to the vacuum initially when the viscosity depends on the density:

$$\left\{ \begin{array}{l} \rho_\tau + (\rho u)_\xi = 0, \\ (\rho u)_\tau + \left(\rho u^2 + P(\rho)\right)_\xi = \left(\mu(\rho) u_\xi\right)_\xi , \end{array} \right. \tag{1.1}$$

in $\tau > 0$, $a(\tau) < \xi < b(\tau)$, where ρ, u and $P(\rho)$ are the density, the velocity and the pressure, respectively, $\mu(\rho) \geq 0$ is the viscosity coefficient, $a(\tau)$ and $b(\tau)$ are the free boundaries, i.e. the interface of the gas and the vacuum:

$$\left\{ \begin{array}{l} \frac{d}{d\tau} a(\tau) = u(a(\tau), \tau), \;\; \frac{d}{d\tau} b(\tau) = u(b(\tau), \tau), \\ \left(-P(\rho) + \mu(\rho) u_\xi\right)(a(\tau), \tau) = 0, \quad \left(-P(\rho) + \mu(\rho) u_\xi\right)(b(\tau), \tau) = 0. \end{array} \right. \tag{1.2}$$

Due to the strong degeneracy at vacuum, both Euler and Navier-Stokes systems for compressible fluids (in which the viscosity is independent of density) behave singularly [7, 10, 16]. In particular, the classical one-dimensional isentropic Navier-Stokes system picks up unphysical solutions for two gases initially separated by vacuum states [7, 10]. To overcome this difficulty, Liu, Xin and Yang in [10] introduced the modified Navier-Stokes system (1.1) in which the viscosity coefficient depends on the density. It is shown in [10] that at least locally in time, the system (1.1) yields the physically relevant solution. As remarked by Liu, Xin and Yang in [10], the model is also motivated by the physical consideration that in the derivation of the compressible Navier-Stokes equations from the Boltzmann equations, the viscosity is not constant and depends on the temperature. For isentrpoic flow, this dependence is translated into the dependence of the viscosity on the density.

For simplicity we consider in this paper

$$\left\{ \begin{array}{l} P(\rho) = A\rho^\gamma, \\ \mu(\rho) = B\rho^\alpha, \end{array} \right. \tag{1.3}$$

where $\gamma > 1$, $A > 0$, $B > 0$, $\alpha > 0$ are constants.

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