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POLYNOMIAL POISSON STRUCTURES ON AFFINE SOLVMANIFOLDS

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A n-dimensional Lie group G equipped with a left invariant symplectic form ω^+ is called a symplectic Lie group. It is well known that ω^+ induces a left invariant affine structure on G. Relative to this affine structure we show that the left invariant Poisson tensor π^+ corresponding to ω^+ is polynomial of degree at most 1 and any right invariant k-multivector field on G is polynomial of degree at most k. If G is unimodular, the symplectic form ω^+ is also polynomial and the volume form $\wedge^{n/2}\omega^+$ is parallel. We show also that any left invariant tensor field on a nilpotent symplectic Lie group is polynomial, in particular, any left invariant Poisson structure on a nilpotent symplectic Lie group is polynomial. Because many symplectic Lie groups admit uniform lattices, we get a large class of polynomial Poisson structures on compact affine solvmanifolds.

1. Introduction and main results

Recall that an affine manifold is a differential manifold M together with a special atlas of coordinate charts such that all coordinate changes extend to affine automorphisms of $\mathbb{R}^n$. These distinguished charts are called affine charts. The data of a flat and torsion free connection ∇ on a manifold M is equivalent to the data of an affine structure.

A tensor field on an affine manifold M is called *polynomial* if in affine coordinates its coefficients are polynomial functions. A Poisson structure on an affine manifold is called polynomial if the space of local polynomial functions is closed under the Poisson bracket. In an equivalent way this means that the associated Poisson bivector is polynomial. For some general results on polynomial tensor fields see [**6, 8–10, 18**]. Let us describe briefly the affine structure associated to a Lie group endowed with a left invariant symplectic form. This affine structure is the context on which we will state