JOURNAL OF
SYMPLECTIC GEOMETRY
Volume 8, Number 4, 359–380, 2010

SYMMETRY OF A SYMPLECTIC TORIC MANIFOLD

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The action of a torus group T on a symplectic toric manifold (M, ω) often extends to an effective action of a (non-abelian) compact Lie group G. We may think of T and G as compact Lie subgroups of the symplectomorphism group $\mathrm{Symp}(M, \omega)$ of (M, ω). On the other hand, (M, ω) is determined by the associated moment polytope P by the result of Delzant [**4**]. Therefore, the group G should be estimated in terms of P or we may say that a *maximal* compact Lie subgroup of $\mathrm{Symp}(M, \omega)$ containing the torus T should be described in terms of P. In this paper, we introduce a root system $R(P)$ associated to P and prove that any irreducible subsystem of $R(P)$ is of type A and the root system $\Delta(G)$ of the group G is a subsystem of $R(P)$ (so that $R(P)$ gives an upper bound for the identity component of G and any irreducible factor of $\Delta(G)$ is of type A). We also introduce a homomorphism $\mathcal{D}$ from the normalizer $N_G(T)$ of T in G to an automorphism group $\mathrm{Aut}(P)$ of P, which detects the connected components of G. Finally, we find a maximal compact Lie subgroup $G_{\max}$ of $\mathrm{Symp}(M, \omega)$ containing the torus T such that $\Delta(G_{\max}) = R(P)$ and $\mathcal{D}$ is onto.

1. Introduction

A *symplectic toric manifold* is a compact connected symplectic manifold (M, ω) with an effective Hamiltonian action of a torus group T of half the dimension of the manifold M. Delzant [**4**] proves that M is equivariantly diffeomorphic to a smooth projective toric variety with the restricted T-action. Moreover he classifies symplectic toric manifolds by showing that the correspondence from symplectic toric manifolds modulo equivalence to their moment polytopes is one-to-one. Therefore, all geometrical information on (M, ω) is encoded in the moment polytope P associated with (M, ω).

The T-action on (M, ω) often extends to an effective action of a (non-abelian) compact Lie group G. We may think of T and G as compact Lie subgroups of the symplectomorphism group $\mathrm{Symp}(M, \omega)$ of (M, ω). Since