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GRID DIAGRAMS AND LEGENDRIAN LENS SPACE LINKS

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Grid diagrams encode useful geometric information about knots in S^3. In particular, they can be used to combinatorially define the knot Floer homology of a knot $K \subset S^3$, and they have a straightforward connection to Legendrian representatives of $K \subset (S^3, \xi_{\text{st}})$, where ξ_{st} is the standard, tight contact structure. The definition of a grid diagram was extended, by Hedden and the authors, to include a description for links in all lens spaces, resulting in a combinatorial description of the knot Floer homology of a knot $K \subset L(p,q)$ for all $p \neq 0$. In the present article, we explore the connection between lens space grid diagrams and the contact topology of a lens space. Our hope is that an understanding of grid diagrams from this point of view will lead to new approaches to the Berge conjecture, which claims to classify all knots in S^3 upon which surgery yields a lens space.

1. Introduction

A grid diagram provides a simple combinatorial means of encoding the data of a link in S^3, as in Figure 1. Although grid diagrams first made an appearance in the late 19th century [**Bru98**], they have enjoyed an abundance of recent attention, due primarily to their connection to contact topology [**Lyo80**, **Cro95**, **Dyn06**, **Mat06**, **OST06**] and combinatorial Heegaard Floer homology [**MOS06**, **MOST06**].

The definition was extended in [**BGH08**] to provide a means of encoding the data of all lens space links via grid diagrams, leading to a combinatorial description of the knot Floer homology of a lens space knot. Figure 2 illustrates the notion of a lens space grid diagram; we delay its precise definition until Section 4.

In the present article, we explore the connection between the contact geometry of a lens space and grid diagrams for lens space links.