JOURNAL OF
SYMPLECTIC GEOMETRY
Volume 7, Number 3, 295–310, 2009

CLASSIFICATION OF MULTIPLICITY FREE HAMILTONIAN ACTIONS OF ALGEBRAIC TORI ON STEIN MANIFOLDS

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A Hamiltonian action of an algebraic torus on a symplectic complex manifold is said to be *multiplicity free* if a general orbit is a lagrangian submanifold. To any multiplicity free Hamiltonian action of an algebraic torus $T \cong (\mathbb{C}^\times)^n$ on a Stein manifold X we assign a certain 5-tuple consisting of a Stein manifold Y, an étale map $Y \to \mathfrak{t}^*$, a set of divisors on Y and elements of $H^2(Y,\mathbb{Z})^{\oplus n}, H^2(Y,\mathbb{C})$. We show that X is uniquely determined by this invariants. Furthermore, we describe all 5-tuples arising in this way.

Contents

1. Introduction

Let X be a smooth manifold with a symplectic form ω and T be a compact torus acting on X by symplectomorphisms. We recall that the action $T : X$ is called *Hamiltonian* if there are $n = \dim T$ functions $H_1, \ldots, H_n \in C^\infty(X)$ such that

(H1) $\{H_i, H_j\} = 0$, where $\{\cdot,\cdot\}$ denotes the Poisson bracket on X induced by ω.