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THE COBORDISM CLASS OF THE MODULI SPACE OF POLYGONS IN $\mathbb{R}^3$

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For any vector $r = (r_1, \dots, r_n)$, let M_r denote the moduli space (under rigid motions) of polygons in $\mathbb{R}^3$ with n-sides whose lengths are $r_1, \dots, r_n$. We give an explicit characterization of the oriented S^1-cobordism class of M_r which depends uniquely on the length vector r.

1. Introduction

The study of the geometry of moduli spaces of polygons with fixed side lengths $r_1, \dots, r_n$ in the Euclidean space has raised, since the 1990s, a remarkable interest in symplectic geometry. These moduli spaces have a very rich structure; they can be described (in two possible ways) as symplectic quotients: see for example [**1**] where Kapovich and Millson show that these spaces are complex-analytic spaces and they define and study the Hamiltonian flows on M_r obtained by bending polygons along diagonals. Another description of M_r as a symplectic reduction is given by Hausmann and Knutson [**2**], who also give a useful geometric interpretation of the bending action.

Let $\mathcal{S}_r = \prod_{j=1}^n S^2_{r_j}$ be the product of n spheres of radii $r_1, \dots, r_n$ respectively; $\mathcal{S}_r$ is a symplectic manifold and a Hamiltonian SO(3)-space with associated moment map

$$\begin{aligned} \mu : \mathcal{S}_r &\longrightarrow \mathrm{Lie}(\mathrm{SO}(3))^* \simeq \mathbb{R}^3 \\ \vec{e} = (e_1, \dots, e_n) &\longmapsto e_1 + \cdots + e_n. \end{aligned}$$

For a (suitably chosen) length vector $r = (r_1, \dots, r_n) \in \mathbb{R}^n_+$ the symplectic quotient $\mathcal{S}_r//\mathrm{SO}(3)$ at the 0-level set is a smooth manifold, and it is defined to be the moduli space M_r [**1**]. Note that the condition $\mu(\vec{e}) = 0$ is the closing condition for a polygon with edge vectors $e_1, \dots, e_n$ starting at an