JOURNAL OF
SYMPLECTIC GEOMETRY
Volume 4, Number 2, 117–148, 2006

# FIBERWISE VOLUME GROWTH VIA LAGRANGIAN INTERSECTIONS

Urs Frauenfelder and Felix Schlenk

We study certain topological entropy-type growth characteristics of Hamiltonian flows of a special form on the cotangent bundle $T^*M$ over a closed Riemannian manifold $(M, g)$. More precisely, the Hamiltonians generating the flows have to be of the form $H(t, q, p) = f(|p|)$ for $|p| \geq 1$, and the growth characteristics reflect how the Riemannian volumes of the unit balls in the fibers of $T^*M$ grow with the flow. Using a version of Lagrangian Floer homology and recent work of Abbondandolo and Schwarz, we obtain uniform lower bounds for the growth characteristics that depend only on $(M, g)$ or, more precisely, on the homology of the sublevel sets of the energy functional defined by the Riemannian metric $g$ on the space of based loops of Sobolev class $W^{1,2}$ in $M$. These lower bounds, in turn, can be estimated from below by purely topological characteristics of $M$. Our results in particular refine previous results of Dinaburg, Gromov, Paternain, and Paternain–Petean on the topological entropy of geodesic flows.

As an application, we obtain that for Riemannian manifolds, all of whose geodesics are closed (so-called $P$-manifolds), the fiberwise volume growth of every symplectomorphism in the symplectic isotopy class of the Dehn–Seidel twist is at least linear. This extends the main result of our paper [**30**] from the class of all currently known $P$-manifolds to all $P$-manifolds.

## 1. Introduction and main results

**1.1. Topological entropy and volume growth.** The topological entropy $h_{\text{top}}(\varphi)$ of a compactly supported $C^1$-diffeomorphism $\varphi$ of a smooth manifold $X$ is a basic numerical invariant measuring the orbit structure complexity of $\varphi$. There are various ways of defining $h_{\text{top}}(\varphi)$, see [**39**]. If $\varphi$ is $C^\infty$-smooth, a geometric way was found by Yomdin and Newhouse in their seminal works [**76**] and [**54**] respectively: Fix a Riemannian metric $g$ on $X$. For $j \in \{1, \dots, \dim X\}$ denote by $\Sigma_j$ the set of smooth compact (not necessarily