JOURNAL OF
SYMPLECTIC GEOMETRY
Volume 9, Number 2, 147–160, 2011

NEGATIVE INFLATION AND STABILITY IN SYMPLECTOMORPHISM GROUPS OF RULED SURFACES

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Consider symplectic ruled surfaces $M^g_\lambda = (\Sigma_g \times S^2, \lambda\sigma_{\Sigma_g} \oplus \sigma_{S^2})$ such that Σ_g has area λ and S^2 has area 1. We show that for $k \geq \lfloor g/2 \rfloor$ the homotopy type of the symplectomorphism groups G^g_λ of M^g_λ is constant as λ increases in the interval $(k, k+1]$, thus generalizing an existent result of Abreu–McDuff for the rational ruled surfaces with $g = 0$. We also investigate the changes in the groups $\pi_* G^g_\lambda$ as λ passes an integer k and show the existence of higher Samelson products in $\pi_{4k+2g} G^g_\lambda$ that exist only for λ in the range $(k, k+1]$. To prove these results we introduce a refinement of the negative inflation technique introduced by Li–Usher.

1. Introduction and results

The purpose of this note is to introduce and apply a refinement of the negative inflation method in a symplectic four-manifold. Inflation was first introduced by Lalonde–McDuff [**13**] for embedded J-holomorphic curves with positive self-intersection and extended later in a weaker version by Li–Usher [**14**] for negative self-intersection curves. Our work is based on the following:

Theorem 1.1. *Fix a symplectic four-manifold (M^4, J, τ_0) such that J is any τ_0-tame almost complex structure. Assume that M admits an embedded J-holomorphic curve $u : (\Sigma, j) \to (M^4, J)$ in a homology class Z with $Z^2 = -m$. For all $\varepsilon > 0$ there exist a family of symplectic forms τ_μ all taming J which satisfy*

$$[\tau_\mu] = [\tau_0] + \mu a_Z$$

for all $0 \leq \mu \leq \frac{\tau_0(Z)}{m} - \varepsilon$, where a_Z is the Poincare dual of Z.

To prove it we adapt and refine McDuff's method [**15**] used in the case of positive curves. In the Li–Usher work, which inspired this paper, the authors prove the above result without the added tameness condition on