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# POSITIVITY OF EQUIVARIANT SCHUBERT CLASSES THROUGH MOMENT MAP DEGENERATION

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For a flag manifold $M = G/B$ with the canonical torus action, the $T$-equivariant cohomology is generated by equivariant Schubert classes, with one class $\tau_u$ for every element $u$ of the Weyl group $W$, and these classes are determined by their restrictions to the fixed point set $M^T \simeq W$. The main result of this article is a positive formula for computing $\tau_u(v)$ in types $A$, $B$, and $C$. We identify $G/B$ with a generic coadjoint orbit and use a result of Goldin and Tolman to compute $\tau_u(v)$ in terms of the induced moment map. Our positive formula, given as a sum indexed by certain saturated chains, follows from a systematic degeneration of the moment map. In type $A$ our formula is equivalent to a classical positive formula that uses summation over certain subwords, but in type C, the two formulas are different.

## Nomenclature

| | |
|---|---|
| $G$ | connected, complex, semisimple Lie group |
| $B$ | Borel subgroup of $G$ |
| $M = G/B$ | flag manifold |
| $T^{\mathbb{C}}$, $T$ | maximal complex torus in $B$ and its compact real form |
| $H_T^*(M) = H_T^*(M; \mathbb{Q})$ | rational $T$-equivariant cohomology of $M$ |
| $\mathfrak{t}$, $\mathfrak{t}^*$ | Lie algebra of $T$ and its dual |
| $\mathcal{B} = \{\alpha_1, \dots, \alpha_n\}$ | simple, positive roots corresponding to $B$ |
| $0 \prec \beta$ | the vector $\beta \in \mathfrak{t}^*$ has non-negative coordinates in $\mathcal{B}$ |
| $\omega_1, \dots, \omega_n$ | fundamental weights corresponding to $\alpha_1, \dots, \alpha_n$ |