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INTEGRALS OF EQUIVARIANT FORMS OVER NON-COMPACT SYMPLECTIC MANIFOLDS

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This article is a result of the AIM workshop on Moment Maps and Surjectivity in Various Geometries (August 9–13, 2004) organized by T. Holm, E. Lerman and S. Tolman. At that workshop I was introduced to the work of Hausel and Proudfoot on hyperkähler quotients [**HP**]. One interesting feature of their article is that they consider integrals of equivariant forms over non-compact symplectic manifolds which do not converge, so they formally *define* these integrals as sums over the zeroes of vector fields, as in the Berline–Vergne localization formula. In this article we introduce a geometric-analytic regularization technique which makes such integrals converge and utilizes the symplectic structure of the manifold. We also prove that the Berline–Vergne localization formula holds for these integrals as well. The key step here is to redefine the collection of integrals $\int_M \alpha(X)$, $X \in \mathfrak{g}$, as a distribution on the Lie algebra $\mathfrak{g}$. We expect our regularization technique to generalize to non-compact group actions extending the results of [**L1**, **L2**].

1. Introduction

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While the definition is perfectly valid, it does not feel satisfactory. The Berline–Vergne localization formula relates a global object (integral of a cohomology class) with a local object (certain quotients defined at zeroes of a vector field). From this point of view, the localization formula is very similar