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COHOMOLOGY OF COURANT ALGEBROIDS WITH SPLIT BASE

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In this paper we study the cohomology $H^\bullet_{\mathrm{st}}(E)$ of a Courant algebroid E. We prove that if E is transitive, $H^\bullet_{\mathrm{st}}(E)$ coincides with the naive cohomology $H^\bullet_{\mathrm{naive}}(E)$ of E as conjectured by Stiénon and Xu. For general Courant algebroids E we define a spectral sequence converging to $H^\bullet_{\mathrm{st}}(E)$. If E is with split base, we prove that there exists a natural transgression homomorphism T_3 (with image in $H^3_{\mathrm{naive}}(E)$) which, together with $H^\bullet_{\mathrm{naive}}(E)$, gives all $H^\bullet_{\mathrm{st}}(E)$. For generalized exact Courant algebroids, we give an explicit formula for T_3 depending only on the Ševera characteristic clas of E.

1. Introduction

The purpose of this paper is to study the cohomology of Courant algebroids. The Courant bracket was first introduced by Courant in 1990 (see [**Cou90**]) in order to describe Dirac manifolds, a generalization of presymplectic and Poisson manifolds. In 1997 Liu, Weinstein and Xu introduced the notion of a Courant algebroid in order to describe Manin triples for Lie bialgebroids ([**LWX97**]). Recently, Courant algebroids have been used as a background to describe generalized complex geometry, see [**Hit03**, **Gua04**] and as target spaces for three-dimensional topological field theory [**Ike01**, **Ike03**, **Par01**, **HP04**, **Roy07**].

Roughly speaking, a Courant algebroid is a pseudo-Euclidean vector bundle $E \to M$ together with an anchor map $\rho : E \to TM$ and a bracket $[.,.]$ on ΓE which satisfy the basic identities, e.g., skew-symmetry, Jacobi identity, Leibniz rule and ad-invariance, only up to anomalies (which are *exact* terms). Up to the anomalies, the bracket and the anchor map are similar to those of a Lie algebroid. Indeed, Courant algebroids appear to be the right framework for pseudo-metric vector bundles equipped with something like a