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GLUING PSEUDOHOLOMORPHIC CURVES ALONG BRANCHED COVERED CYLINDERS II

Michael Hutchings and Clifford Henry Taubes

This paper and its prequel ("Part I") prove a generalization of the usual gluing theorem for two index 1 pseudoholomorphic curves U_+ and U_- in the symplectization of a contact 3-manifold. We assume that for each embedded Reeb orbit γ, the total multiplicity of the negative ends of U_+ at covers of γ agrees with the total multiplicity of the positive ends of U_- at covers of γ. However, unlike in the usual gluing story, here the individual multiplicities are allowed to differ. In this situation, one can often glue U_+ and U_- to an index 2 curve by inserting genus zero branched covers of $\mathbb{R}$-invariant cylinders between them. This paper shows that the signed count of such gluings equals a signed count of zeroes of a certain section of an obstruction bundle over the moduli space of branched covers of the cylinder. Part I obtained a combinatorial formula for the latter count and, assuming the result of the present paper, deduced that the differential ∂ in embedded contact homology satisfies $\partial^2 = 0$. The present paper completes all of the analysis that was needed in Part I. The gluing technique explained here is in principle applicable to more gluing problems. We also prove some lemmas concerning the generic behavior of pseudoholomorphic curves in symplectizations, which may be of independent interest.

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