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# SYMPLECTIC FIELD THEORY AND QUANTUM BACKGROUNDS

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We derive an $L_\infty$ structure associated to a polarized quantum background and characterize the obstructions to finding a versal solution to the quantum master equation (QME). We illustrate how symplectic field theory (SFT) is an example of a polarized quantum background and discuss the $L_\infty$ structure in the SFT context. The discussion may be summarized as follows: given a contact manifold $M$ with contact homology $\mathcal{H}$, one can define an $L_\infty$ algebra on $\mathcal{A}[[\hbar]]$, where $\mathcal{A} = \mathcal{A}(M)$ is the free symmetric algebra on the vector space of Reeb orbits of $M$. The obstructions to finding a versal solution to the QME in $\mathcal{A}[[\hbar]]$ are organized into what we call the kappa invariant, which is a new differential $\kappa : \mathcal{H}[[\hbar]] \to \mathcal{H}[[\hbar]]$.

Also, a quantum background associated to an arbitrary manifold is defined, which does not use any contact structure. It agrees with the one from SFT of the unit cotangent bundle of the manifold in some cases, but might, in general, be different.

## 1. Introduction

Since the celebrated Gromov compactness result for $J$-holomorphic curves in symplectic manifolds [**7**], symplectic geometers have been using these curves to study symplectic and contact manifolds, as well as questions in other fields connected to these manifolds. The compactness of these curves is often restated in some form of the equation

$$\partial X = X * X.$$

Here $X$ is some collections of moduli spaces of these curves, $\partial$ is the topological boundary of these spaces, and $*$ embodies the (possibly self-) gluing of the $J$-holomorphic curves as they appear in Gromov's sequential limit. A common goal is to develop algebraic machinery (and its subsequent