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# ON THE SYMPLECTIC FORM OF THE MODULI SPACE OF PROJECTIVE STRUCTURES

Pablo Arés-Gastesi and Indranil Biswas

Let $S$ be a $C^\infty$ compact connected oriented surface whose genus is at least two. Let $\mathcal{P}(S)$ be the moduli space of isotopic classes of projective structures associated to $S$. The natural holomorphic symplectic form on $\mathcal{P}(S)$ will be denoted by $\Omega_P$. The natural holomorphic symplectic form on the holomorphic cotangent bundle $T^*\mathcal{T}(S)$ of the Teichmüller space $\mathcal{T}(S)$ associated to $S$ will be denoted by $\Omega_T$. Let $e : \mathcal{T}(S) \longrightarrow \mathcal{P}(S)$ be the holomorphic section of the canonical holomorphic projection $\mathcal{P}(S) \longrightarrow \mathcal{T}(S)$, given by the Earle uniformization. Let $T_e : T^*\mathcal{T}(S) \longrightarrow \mathcal{P}(S)$ be the biholomorphism constructed using the section $e$. We prove that $T_e^*\Omega_P = \pi \cdot \Omega_T$. This remains true if $e$ is replaced by a large class of sections that include the one given by the Schottky uniformization.

## 1. Introduction

A projective structure on a smooth compact connected oriented surface $S$ is defined by giving a covering of $S$ by coordinate charts, where the coordinate functions are orientation preserving diffeomorphisms to open subsets of $\mathbb{C}$, such that all the transition functions are Möbius transformations. Two projective structures are called equivalent if they differ by a diffeomorphism of $S$ homotopic to the identity map. Let $\mathcal{P}(S)$ denote the equivalence classes of projective structures on $S$.

The Teichmüller space $\mathcal{T}(S)$ for $S$ parametrizes all the equivalence classes of complex structures on $S$ compatible with its orientation; two complex structures are called equivalent if they differ by a diffeomorphism of $S$ homotopic to the identity map. Both $\mathcal{P}(S)$ and $\mathcal{T}(S)$ are complex manifolds,