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OZSVÁTH–SZABÓ INVARIANTS AND TIGHT CONTACT 3-MANIFOLDS, III

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We characterize L-spaces which are Seifert fibered over the 2-sphere in terms of taut foliations, transverse foliations and transverse contact structures. We give a sufficient condition for certain contact Seifert fibered 3-manifolds with $e_0 = -1$ to have nonzero contact Ozsváth–Szabó invariants. This yields an algorithm for deciding whether a given small Seifert fibered L-space carries a contact structure with nonvanishing contact Ozsváth–Szabó invariant. As an application, we prove the existence of tight contact structures on some 3-manifolds obtained by integral surgery along a positive torus knot in the 3-sphere. Finally, we prove planarity of every contact structure on small Seifert fibered L-spaces with $e_0 \geq -1$, and we discuss some consequences.

1. Introduction

The Ozsváth–Szabó homology groups of a closed, oriented 3-manifold Y [**36**, **37**] capture important topological information about Y. For example, by [**35**, Theorem 1.1] the Thurston semi-norm is determined by the evaluation of the first Chern classes of spin^c structures with nontrivial Ozsváth–Szabó homology groups. For rational homology spheres, however, the Thurston norm is trivial, while the Ozsváth–Szabó homology groups are special: the group $\widehat{\mathrm{HF}}(Y, \mathbf{t})$ has odd rank for each spin^c structure $\mathbf{t} \in \mathrm{Spin}^c(Y)$. A rational homology sphere Y shows the simplest possible Heegaard Floer-theoretic behavior if for every spin^c structure $\mathbf{t} \in \mathrm{Spin}^c(Y)$ the group $\widehat{\mathrm{HF}}(Y, \mathbf{t})$ with integral coefficients is isomorphic to $\mathbb{Z}$, in which case Y is called an *L-space*. In the present paper, we shall always use $\mathbb{Z}/2\mathbb{Z}$–coefficients. In this case, if Y is an L-space then $\widehat{\mathrm{HF}}(Y, \mathbf{t}) \cong \mathbb{Z}/2\mathbb{Z}$ for every $\mathbf{t} \in \mathrm{Spin}^c(Y)$. Since $\widehat{\mathrm{HF}}(Y, \mathbf{t}) \cong \widehat{\mathrm{HF}}(-Y, \mathbf{t})$ for each $\mathbf{t} \in \mathrm{Spin}^c(Y)$, the 3-manifold Y is an L-space if and only if $-Y$ is.