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THE GROUP OF HAMILTONIAN HOMEOMORPHISMS AND C^0-SYMPLECTIC TOPOLOGY

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The main purpose of this paper is to carry out some of the foundational study of C^0-*Hamiltonian geometry* and C^0-*symplectic topology*. We introduce the notion of *Hamiltonian topology* on the space of Hamiltonian paths and on the group of Hamiltonian diffeomorphisms. We then define the *group*, denoted by $\mathrm{Hameo}(M, \omega)$, consisting of *Hamiltonian homeomorphisms* such that

$$\mathrm{Ham}(M, \omega) \subsetneq \mathrm{Hameo}(M, \omega) \subset \mathrm{Sympeo}(M, \omega),$$

where $\mathrm{Sympeo}(M, \omega)$ is the *group* of *symplectic homeomorphisms*. We prove $\mathrm{Hameo}(M, \omega)$ is a *normal subgroup* of $\mathrm{Sympeo}(M, \omega)$ and contains all the time-one maps of Hamiltonian vector fields of $C^{1,1}$-functions, and $\mathrm{Hameo}(M, \omega)$ is path-connected and so contained in the identity component $\mathrm{Sympeo}_0(M, \omega)$ of $\mathrm{Sympeo}(M, \omega)$.

We also prove that the *mass flow* of any Hamiltonian homeomorphism vanishes. In the case of a closed orientable surface, this implies that $\mathrm{Hameo}(M, \omega)$ is strictly smaller than the identity component of the group of area-preserving homeomorphisms when $M \neq S^2$. For $M = S^2$, we conjecture that $\mathrm{Hameo}(S^2, \omega)$ is still a proper subgroup of $\mathrm{Sympeo}_0(S^2, \omega)$.

Dedicated to Dusa McDuff

1. Introduction

Let (M, ω) be a connected symplectic manifold. *Unless explicit mention is made to the contrary, M will be closed.* See Section 6 for the necessary changes in the non-compact case or in the case with boundary. Denote by $\mathrm{Symp}(M, \omega)$ the group of symplectic diffeomorphisms i.e., the subgroup of $\mathrm{Diff}(M)$ consisting of diffeomorphisms $\phi : M \to M$ such that $\phi^*\omega = \omega$. We equip $\mathrm{Diff}(M)$ with the C^∞-topology. Then $\mathrm{Symp}(M, \omega)$ forms a closed topological subgroup. We call the induced