JOURNAL OF
SYMPLECTIC GEOMETRY
Volume 5, Number 1, 43–137, 2007

GLUING PSEUDOHOLOMORPHIC CURVES ALONG BRANCHED COVERED CYLINDERS I

Michael Hutchings and Clifford Henry Taubes

This paper and its sequel prove a generalization of the usual gluing theorem for two index $\mathbf{1}$ pseudoholomorphic curves $\boldsymbol{u}_+$ and $\boldsymbol{u}_-$ in the symplectization of a contact 3-manifold. We assume that for each embedded Reeb orbit $\boldsymbol{\gamma}$, the total multiplicity of the negative ends of $\boldsymbol{u}_+$ at covers of $\boldsymbol{\gamma}$ agrees with the total multiplicity of the positive ends of $\boldsymbol{u}_-$ at covers of $\boldsymbol{\gamma}$. However, unlike in the usual gluing story, here the individual multiplicities are allowed to differ. In this situation, one can often glue $\boldsymbol{u}_+$ and $\boldsymbol{u}_-$ to an index $\mathbf{2}$ curve by inserting genus zero branched covers of $\mathbb{R}$-invariant cylinders between them. We establish a combinatorial formula for the signed count of such gluings. As an application, we deduce that the differential $\boldsymbol{\partial}$ in embedded contact homology satisfies $\boldsymbol{\partial}^{\mathbf{2}} = \mathbf{0}$.

This paper explains the more algebraic aspects of the story, and proves the above formulas using some analytical results from part II.

Dedicated to Dusa McDuff

Contents