

GLUING PSEUDOHOLOMORPHIC CURVES ALONG BRANCHED COVERED CYLINDERS I

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This paper and its sequel prove a generalization of the usual gluing theorem for two index **1** pseudoholomorphic curves \mathbf{u}_+ and \mathbf{u}_- in the symplectization of a contact 3-manifold. We assume that for each embedded Reeb orbit γ , the total multiplicity of the negative ends of \mathbf{u}_+ at covers of γ agrees with the total multiplicity of the positive ends of \mathbf{u}_- at covers of γ . However, unlike in the usual gluing story, here the individual multiplicities are allowed to differ. In this situation, one can often glue \mathbf{u}_+ and \mathbf{u}_- to an index **2** curve by inserting genus zero branched covers of \mathbb{R} -invariant cylinders between them. We establish a combinatorial formula for the signed count of such gluings. As an application, we deduce that the differential ∂ in embedded contact homology satisfies $\partial^2 = 0$.

This paper explains the more algebraic aspects of the story, and proves the above formulas using some analytical results from part II.

Dedicated to Dusa McDuff

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