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THE FUNDAMENTAL GROUP OF SYMPLECTIC MANIFOLDS WITH HAMILTONIAN LIE GROUP ACTIONS

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Let (M, ω) be a connected, compact symplectic manifold equipped with a Hamiltonian G action, where G is a connected compact Lie group. Let ϕ be the moment map. In [**12**], we proved the following result for $G = S^1$ action: as fundamental groups of topological spaces, $\pi_1(M) \cong \pi_1(M_{\text{red}})$, where M_{red} is the symplectic quotient at any value of the moment map ϕ, and $\cong$ denotes "isomorphic to". In this paper, we generalize this result to other connected compact Lie group G actions. We also prove that the above fundamental group is isomorphic to that of M/G. We briefly discuss the generalization of the first part of the results to non-compact manifolds with proper moment maps.

1. Introduction

Let (M, ω) be a connected, compact symplectic manifold. Let us assume a connected compact Lie group G acts on M in a Hamiltonian fashion with moment map $\phi : M \to \mathfrak{g}^*$, where $\mathfrak{g}^*$ is the dual Lie algebra of G. Assume ϕ is equivariant with respect to the G action, where G acts on $\mathfrak{g}^*$ by the co-adjoint action. Take a moment map value a in $\mathfrak{g}^*$, the space $M_{G\cdot a} = \phi^{-1}(G\cdot a)/G$ is called the symplectic quotient or the reduced space at the co-adjoint orbit $G\cdot a$. If G_a is the stabilizer group of a under the co-adjoint action, by equivariance of the moment map, the two reduced spaces are equal: $M_a = \phi^{-1}(a)/G_a = \phi^{-1}(G\cdot a)/G = M_{G\cdot a}$. We will use the two notations interchangeably. The space $M_{G\cdot a}$ can be a smooth symplectic manifold, or a symplectic orbifold, or a symplectic stratified space. The