

GROUPOIDS, BRANCHED MANIFOLDS AND MULTISECTIONS

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Cieliebak *et al.* recently formulated a definition of branched submanifold of Euclidean space in connection with their discussion of multivalued sections and the Euler class. This note proposes an intrinsic definition of a weighted branched manifold \underline{Z} that is obtained from the usual definition of oriented orbifold groupoid by relaxing the properness condition and adding a weighting. We show that if \underline{Z} is compact, finite dimensional and oriented, then it carries a fundamental class $[\underline{Z}]$. Adapting a construction of Liu and Tian, we also show that the fundamental class $[\underline{X}]$ of any oriented orbifold \underline{X} may be represented by a map $\underline{Z} \rightarrow \underline{X}$, where the branched manifold \underline{Z} is unique up to a natural equivalence relation. This gives further insight into the structure of the virtual moduli cycle in the new polyfold theory recently constructed by Hofer *et al.*

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