JOURNAL OF
SYMPLECTIC GEOMETRY
Volume 3, Number 1, 17–54, 2005

LOCALIZATION FOR THE NORM-SQUARE OF THE MOMENT MAP AND THE TWO-DIMENSIONAL YANG–MILLS INTEGRAL

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1. Introduction

The first seven sections of the paper contain a version of localization for the norm-square of the moment map in equivariant de Rham theory. The main Theorem 5.1 expresses the push-forward of an equivariant cohomology class on a Hamiltonian K-manifold with proper moment map as a sum of contributions from fixed point components of one-parameter subgroups corresponding to the critical values of the norm-square of the moment map. If the critical set of the norm-square is non-degenerate in a sense explained below, there is an improved result Theorem 6.1 which expresses the push-forward as an integral over the quotient of the critical set. Many of the ingredients appear in the papers of Paradan [**35**, **36**, **37**], who proved a version of the same result. The proof given here is different from Paradan's. The existence of a localization formula, but not the precise form of the contributions, was first suggested by Witten [**47**] in his study of two-dimensional Yang–Mills theory. Later Jeffrey and Kirwan [**20**] gave a formula which had a similar purpose but was expressed in terms of rather different fixed point data. A K-theory version was given by Vergne [**45**] and Paradan [**37**]. A similar result for sheaf cohomology that I learned from Teleman is explained in the eighth section. This paper arose out of an attempt to bring the theory for de Rham cohomology to the same level. The attempt was not entirely successful, mainly because the formula depends on the existence of push-forwards over non-compact manifolds which are more naturally defined in sheaf cohomology.

The ninth section contains a definition and computation of the Yang–Mills path integral in two dimensions. The idea is to reverse the logic in Witten's [**47**] paper and take the "stationary phase approximation" (that is, the localization formula) as the definition of the path integral. In order