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BOOK REVIEW

A New Approach to Differential Geometry Using Clifford's Geometric Algebra, by John Snygg, Springer, New York, 2012, xvii + 465pp., ISBN 978-0-8176-8283-8.

The book under review is perfectly organized textbook for undergraduate students in mathematics and physics due to the large experience of the author. Based on the matrix representation of the Clifford's algebra it provides an effective and universal algebraic formalism for studying the classical differential geometry of curves and surfaces and differential geometry of Riemannian and Pseudo-Riemannian manifolds. More precisely Chapters 2,3 and 4 are devoted to the matrix interpretation of the Clifford's algebra in the Euclidean three-space, Minkowski four-space and flat n-space respectively. The key point is the construction of geometrically arising basis of three involutive and skew-commuting matrices formig a subspace in the space of the symmetric 4×4 matrices, which is isomorphic to the Euclidean three-space E^3. This basis consists of the matrices, which can be written as

$$\mathrm{e}_1 = \begin{pmatrix} \mathrm{O} & \vdots & \mathrm{L} \\ \cdots & & \cdots \\ \mathrm{L} & \vdots & \mathrm{O} \end{pmatrix}, \qquad \mathrm{e}_2 = \begin{pmatrix} \mathrm{O} & \vdots & \mathrm{M} \\ \cdots & & \cdots \\ \mathrm{M} & \vdots & \mathrm{O} \end{pmatrix}, \qquad \mathrm{e}_3 = \begin{pmatrix} \mathrm{I} & \vdots & \mathrm{O} \\ \cdots & & \cdots \\ \mathrm{O} & \vdots & -\mathrm{I} \end{pmatrix}.$$

Here $\mathrm{L} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\mathrm{M} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ are the well known matrices of reflections, I is the unit and O is the zero 2×2 matrices. Because of the above and

$$\mathrm{L}^2 = \mathrm{M}^2 = \mathrm{I}, \quad \mathrm{ML} = -\mathrm{LM} = \mathrm{J}, \quad \mathrm{J}^2 = -\mathrm{I} \quad \text{then} \quad \mathrm{e}_s^2 = \mathrm{I}, \quad \mathrm{e}_s\mathrm{e}_k = -\mathrm{e}_k\mathrm{e}_s.$$

The matrices $\{\mathrm{L}, -\mathrm{iJ}, \mathrm{M}, \mathrm{i} = \sqrt{-1}\}$ are usually denoted as $\{\sigma_1, \sigma_2, \sigma_3\}$ which is the well known triple of the Pauli spin-matrices. Thus quaternions, octonions (closely related to the rotations of unit spheres), the set of Dirac vectors, p-vectors and q-forms take natural place in this matrix approach. All real linear combinations of the 4×4-matrices $\mathrm{I}, \mathrm{e}_s, \mathrm{e}_s\mathrm{e}_k, \mathrm{e}_s\mathrm{e}_k\mathrm{e}_t,\ \mathrm{e}_{i_1}\mathrm{e}_{i_2}\ldots\mathrm{e}_{i_p}$, (usually called p-vectors) represent the Clifford numbers and generate Clifford's algebra over the field of reals (the only considered case in this text). The last one has a natural