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BOOK REVIEW

Index Theory – With Applications to Mathematics and Physics, by David D. Bleecker and Bernhelm Booß-Bavnbeck, International Press, Somerville 2013. xxii+769 pp. ISBN: 978-1-57146-264-0 58-02.

The index theory appears to be a broad and deep theory connecting several mathematical disciplines, which has a significant impact on many areas of physics. Its development initiated a development of a part of K-theory within differential geometry and topology. The monograph "Index Theory with Applications to Mathematics and Physics" contains a thorough description of the index theory and covers a plenty of applications of this theory in the mathematics of PDE's, manifolds, vector bundles, and in theoretical and mathematical physics.

Since the authors track the index theory as it was developed quite faithfully, we can acquaint the content of the book to readers as follows.

The content of the individual chapters will be described in a more detail after that. The index of a linear operator measures the gap between the null space of the operator and the space of elements in the target of the operator which cannot be achieved by the operator (the cokernel). In symbols, if $L : V \to W$ is a map between two k-linear vector spaces, its index is the number $\dim_k \ker L - \dim_k \operatorname{coker} L$ where the cokernel is defined by $\operatorname{coker} L = W/\operatorname{im} L$. In a more abstract way, it can be considered as the element $\ker L - \operatorname{coker} L$ of the K-group of k (k is considered as a ring). If the vector spaces V and W are finite dimensional, the index does not depend on the map L and equals the dimension of V minus the dimension of the target W. Thus the index has a meaning in infinite dimension only.

One of the first question which arises is the following one. For which (infinite dimensional) vector spaces and which (bounded) operators the index may be considered? As the authors do and as it seems to be common, one usually considers the source and target space to be Hilbert spaces. Let us recall that a Fredholm operator is a bounded operator between Hilbert spaces (or Banach spaces eventually) for which the dimensions of its kernel and cokernel are finite – the number index is well defined, and such that its image is closed – the formal subtraction makes