JGSP **2021** (2011) 95–98

JOURNAL OF
Geometry and Symmetry in Physics

BOOK REVIEW

Elementary Differential Geometry, by Andrew Pressley, Springer, London 2010, xi+473 pp., ISBN 978-1-84882-890-2.

The book is devoted to the differential geometry of curves and surfaces in the three-dimensional Euclidean space and consists of thirteen chapters.

First Chapter - *Curves in the Plane and in Space*, is split into five sections. At first two types of curves are discussed - level curves and parameterized curves, as well the relation between them. For a smooth parameterized curve $\gamma : (\alpha, \beta) \to \mathbb{R}^n, n \geqq 2, -\infty \leqq \alpha < \beta \leqq \infty$, the author introduces the notions: the tangent vector $\dot{\gamma}(t)$ at the point $\gamma(t)$, an arc length $s(t) = \int_{t_0}^{t} ||\dot{\gamma}(u)|| \mathrm{d}u$ starting at the point $\gamma(t_0)$, the speed $||\dot{\gamma}(t)||$ of the point $\gamma(t)$ and reparametrization. The following objects are also defined: a unit-speed, regular, periodic and closed curves. It is proved that a parametrized curve has a unit-speed reparametrization if and only if it is regular.

Chapter two - *How Much Does a Curve Curve*, is structured into three sections. This chapter contains the following themes of a regular curve: curvature κ (measuring the extent to which a curve which is not contained in a straight line curve), torsion τ (measuring the extent to which a curve in $\mathbb{R}^3$ is not contained in a plane curve in $\mathbb{R}^n, n \geqq 2$), signed unit, normal n_s and signed curvature κ_s of a plane curve, total signed curvature $\int_0^l \kappa_s \mathrm{d}s$ of a closed curve of length l, a turning angle $\varphi(s)$ of a plane curve, Frenet-Serret equations of a curve in $\mathbb{R}^3$. It is proved that: $\kappa_s = \mathrm{d}\varphi/\mathrm{d}s$ (i.e., the signed curvature is the rate of which the tangent vector of the curve rotates) and the theorem that the curvature κ and the torsion τ (for a curve in $\mathbb{R}^3$) determine the curve up to a motion in the space.

Chapter three - *Global Properties of Curves*, is split into three sections. Here are presented (without complete proofs) the following global results for a simple closed curve γ - the Hopf's Umlaufsatz that the total signed curvature of γ is $\pm 2\pi$, the isoperimetric inequali $A(\gamma) \leqq 1/4\pi l(\gamma)^2$, where $A(\gamma)$ is the area contained by γ and $l(\gamma)$ is the length of γ and the Four Vertex Theorem.

Chapter four - *Surfaces in Three Dimensions*, is structured into five sections. A subset S of $\mathbb{R}^3$ is called a surface if for every point $p \in S$ there is an open set U