JGSP **12** (2008) 97–100

JOURNAL OF
Geometry and Symmetry
in Physics

BOOK REVIEW

Compactifications of Symmetric and Locally Symmetric Spaces, by Armand Borel and Lizhen Ji, Birkhäuser, Boston, 2006, xx + 479pp., ISBN 100-8176-3247-6.

The book under review is a survey on the compactifications of the symmetric spaces and locally symmetric spaces, analyzing their properties, relationships and providing uniform methods for their construction. The basic ideas are extracted, formulated clearly and illustrated on various examples. The first part is devoted to the compactifications $\overline{X}$ of the symmetric spaces X, containing X as a dense open subset. The second one provides smooth compact manifolds M, containing finite disjoint unions of symmetric spaces, embedded as open but not dense subsets. The closure of each symmetric space in M is a manifold with corners. The third part deals with the compactifications of the locally symmetric spaces, their metric and spectral properties. The second part is mostly written by Armand Borel before his death on August 11, 2003. The rest of the book is worked out by Lizhen Ji.

Chapter 1 recalls the original motivations, constructions, properties and applications of the geodesic compactification $X \cup X(\infty)$; Karpelevič compactification $\overline{X}^K$; Satake compactifications $\overline{X}^S_\tau$, associated with certain irreducible projective representations $\tau : G \to P\mathrm{SL}(n, \mathbb{C})$ of the isometry group G of X; Baily-Borel compactification $\overline{X}^{BB}$; Frustenberg compactifications $\overline{X}^F_I$, corresponding to subsets I of simple roots of a fixed minimal parabolic subgroup $P_o \subset G$ with respect to its split component A_{P_o} and Martin compactifications $X \cup \partial_\lambda X$ for $\lambda \leq \lambda_o(X)$, where $\lambda_o(X)$ stands for the bottom of the spectrum of the Laplacian Δ on X.

The second chapter provides a uniform construction of the aforementioned compactifications. It is intrinsic in the sense that does not use closures under embeddings in compact spaces. This approach relates the compactifications of the symmetric spaces X with the compactifications of their corresponding locally symmetric spaces $\Gamma \setminus X$. For a suitable collection $\mathcal{C}$ of parabolic subgroups $P \subset G$, which is invariant under the adjoint action of G, one defines $\overline{X} = X \cup \coprod_{P \in \mathcal{C}} e(P)$ with boundary components $e(P)$, depending on the horospherical decomposition of X with respect to P or its refinements. The topology of $\overline{X}$ is given by the con-