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BOOK REVIEW

Lie-Bäcklund-Darboux Transformations by Charles Li and Artyom Yurov, International Press, Somerville 2014, ix+160pp, ISBN: 978-1-57146-288-6.

The Bäcklund-Darboux Transformations (BDT) are named after the pioneering works of Bäcklund and Darboux published at the end of 19-th century considering the sine-Gordon equation which at that time was related only to issues in classical differential geometry of surfaces. They introduced a procedure to find a new solution starting from a given one. The name Bäcklund transformation sometimes is attached to systems of functional equations involving function u and v in such a way that if u satisfies a given fixed partial differential equation the function v satisfies another fixed partial differential equation. The name Darboux transformation is then used in case the two partial differential equations are the same. The functional equation often contains the so-called 'spectral parameter' which importance has been understood much later as relations with some other classical topics in the spectral theory like the Crum-Krein formulas, insertion of additional bound states etc.

Interestingly, the issue, after being a field of active research for some period, gradually has been forgotten and came back again after the discovery in the 70-ties of the past century of the so called completely integrable nonlinear partial differential equations such as now famous Korteweg-de Vries equation and the nonlinear Schrödinger equation. The sine-Gordon equation turned out to be completely integrable also. The new technique applied to solve the completely integrable equations, known as Inverse Scattering Method (ISM) has been intensively developed in the past decades and remains a field of active research until now. The ISM for solving a given Partial differential equation involves a spectral problem $L\psi = 0$ (auxiliary linear problem) such that the differential equation to be solved could be written as compatibility condition $[L, A] = 0$ between the spectral problem $L\psi = 0$ and another spectral problem $A\psi = 0$. The representation of the given