JGSP **36** (2014) 117–119

JOURNAL OF
Geometry and Symmetry in Physics
ISSN 1312-5192

BOOK REVIEW

The Defocusing NLS Equation and Its Normal Form by Benoît Grébert and Thomas Kappeler, European Mathematical Society, Zürich 2014, x+166pp, ISBN: 978-3-03719-131-6.

The authors are famous mathematicians. Professor Thomas Kappeler is a Director of the Institute for Mathematics at University of Zürich, Switzerland. Professor Benoit Grebert is a Director of the Jean Leray Laboratory of Mathematics at the University of Nantes, France.

Their book is dedicated to the analysis of the defocusing nonlinear Schrödinger equation (NLS)

$$\mathrm{i}\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} - 2|u|^2 u = 0. \tag{1}$$

The authors claim that the same methods can be applied also to the focusing NLS equations, which is different from (1) only in the sign of the nonlinear term

$$\mathrm{i}\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} + 2|u|^2 u = 0. \tag{2}$$

In order to relate to each of this equations a valid mathematical problem one needs to specify first the class of functions $\mathcal{M}$ to which the solution $u(x,t)$ should belong. From this point of view there are three different classes of solutions: i) the class of smooth functions $\mathcal{M}_1$ vanishing fast enough for $x \to \pm\infty$; ii) the class of smooth functions $\mathcal{M}_2$ tending fast enough to constant for $x \to \pm\infty$, i.e. $\lim_{x\to\pm\infty} u(x,t) = \rho \mathrm{e}^{\pm i\phi_0}$; iii) the class of smooth periodic functions $\mathcal{M}_3$.

All three types of problems have their specific peculiarities. In a sense, the periodic problem is the most general one; the other two can be obtained as limiting cases of it [1]. It is only natural that the best studied cases are i) and ii) for the focusing NLS. An important reason for that is that it has a number of important physical applications [1, 2].

Both equations have many features in common. First of all, they both allow Lax representation [2, 3]. This means that one can apply to both of them the inverse