

Geometry and Symmetry in Physics

BOOK REVIEW

Structure and Geometry of Lie Groups, by Joachim Hilgert and Karl-Hermann Neeb, Springer Monographs in Mathematics, Springer, New York 2012, x+744 pp. ISBN: 978-0-387-84793-1 (Print) 978-0-387-84794-8 (Online).

The theory of Lie groups and Lie algebras lies at the heart of modern-day mathematics, with applications too numerous to mention. Not surprisingly, the literature abounds with texts on the subject, ranging from the introductory to treatises beyond and well beyond an introduction.

Many beginner texts, geared towards students, offer shortcuts into subjects such as a Semi-Simple Representation Theory or Complex Geometry, thereby evading intricacies of necessity. Other, more specialised tracts focus by definition on particular species of Lie groups – nilpotent, solvable, or semi-simple – and on questions specific to that lineage. This state of the literature is perfectly efficient and satisfactory to the novice and the expert. However, in the middle ground connecting these boundaries of the community, a compendium was lacking that was comprehensive without resorting to generalities.

As the title suggests, the present monograph addresses the topic from such a general vantage point: combining a hands-on initiation *via* matrix groups with the thorough treatment of the general structure theory of Lie algebras and the machinery of Differential Geometry, it culminates in the definitive study of the most salient structure theoretical questions – decompositions, conjugacy of tori and maximal compacts, closedness of subgroups, linearity, complexifications, disconnectedness, and covering theory.

Thus, the aim is to provide both, on one hand, a serious introduction to Lie theory, placing the neophyte on solid footing, and on the other, a profound manual to the fine points of structure theory, which any practioner of Lie groups is sure to stumble over, and for which even the expert may be in want of a reference. Further developments and applications are intentionally left to the existing literature.

As the authors state in the *Preface*, the present work is based on [3]. The latter book followed a not entirely different idea: Namely, to give at once a no-nonsense introduction to Lie theory and a presentation of the full structure theory in all its