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BOOK REVIEW

Geometric Mechanics and Symmetry. From Finite to Infinite Dimensions, by Darryl D. Holm, Tanya Schmah and Cristina Stoica, Oxford University Press, Oxford, 2009, xi + 515 pp., ISBN: 978-0-19-921290-3

The purpose of the book is to provide the unifying viewpoint of Lagrangian and Hamiltonian mechanics in the coordinate-free language of differential geometry in the spirit of the Marsden-Ratiu school. The book is similar in content - although less formal - to the book by J. Marsden and T. Ratiu [7]. One can also mention the companion two-volumes book by Holm [4, 5] written at a more basic level, and that one can recommend as an introductory reading. The classical treatises on the subject are the books by Abraham-Marsden [1], Arnold [2] and Libermann-Marle [6].

Typical applications are N-particle systems, rigid bodies, continua such as fluids and electromagnetic systems that illustrate the powerfulness of the adopted point of view. The geometrical structure allows the covering of both the finite-dimensional conservative case (first part of the book) and the infinite dimensional situation in the second part. The notion of symmetry here is central, as it allows a reduction of the number of dimensions of the mechanical systems, and further exploits the conserved quantities (momentum map) associated to symmetry. Lie group symmetries, Poisson reduction and momentum maps are first discussed.

The concepts are introduced in a progressive and clear manner in the first part of the book. The second part devoted to infinite dimensional systems is motivated by the identification of Euler's ideal fluid motion with the geodesic flow on the group of volume-preserving diffeomorphism. The Euler-Poincaré (EP) variational principle for the Euler fluid equations is exposed in the framework of geometric mechanics, in association with Lie-Poisson Hamiltonian structure of Noether's theorem and momentum maps. Original applications of the Euler-Poincaré equations to solitons, computational anatomy, image matching, or geophysical fluid dynamics are given at the end of the second part of the book.

Here the first chapter recapitulates the Newtonian, Lagrangian and Hamiltonian