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# BOOK REVIEW

*Lie Groups, Physics, and Geometry*, by Robert Gilmore, Cambridge University Press, Cambridge, 2008, ix + 319pp., ISBN 978-0-521-88400-6.

This is the second book on the Lie groups and Lie algebras by R. Gilmore who is a well-known expert in this field. It is aimed as an introduction to the subject for physicists, engineers and chemists and was intended as a shrunk and appropriately restructured revision of his first book [1] with some accents on the most useful for physicists (and students) aspects, avoiding too technical developments.

In the first chapter the author presents the elements of the Galois theory by describing how the structure of the discrete symmetry group of a polynomial equation determines whether the equation is solvable in radicals and notes that Galois theory motivated Marius Sophus Lie to develop analogous theory for the ordinary differential equations.

In the second chapter the basic algebraic and topological properties of Lie groups are listed and illustrated via the unimodular group $\mathrm{SL}(2,\mathbb{R})$. The third chapter is devoted to the matrix groups $\mathrm{GL}(\mathrm{n},\mathbb{F})$, where $\mathbb{F}$ could be any of the field of real/complex numbers or quaternions and their various subgroups. The author underlines that most of the Lie groups used in applied mathematics and physics are matrix groups, matrix groups over finite fields being however out of the scope of the book.

Chapters 4 and 5 are devoted to the study of Lie algebras. The discussion begins with the explanation that the Lie algebras are useful as they linearize the Lie group in the neighborhood of any of its point. The study of the Lie algebra is easier since it is a linear vector space and one can use all available standard tools for such spaces. It is underlined also that the Lie algebra of a given group retains most but not all of its properties. The inverse operation, called the EXPonentiation, maps (locally) the Lie algebra to the group manifold and parameterizes the Lie group. Basic structure properties and notions of Lie algebras are introduced, such as structure constants, Hilbert-Schmidt and Cartan-Killing inner products and the related invariant metric and measure on the group manifold. The group $\mathrm{SL}(2,\mathbb{R})$ is