JGSP **11** (2008) 97–99

JOURNAL OF
Geometry and Symmetry in Physics

BOOK REVIEW

Lectures on Kähler Manifolds, by Werner Ballmann, European Mathematical Society, 2006, viii + 172pp., ISBN 978-3-03719-025-8.

This is an advanced book on Kähler geometry based on the lectures of a course delivered at University of Bonn and Erwin Schrödinger Institute in Wien. The author is a well-known name in Riemannian geometry.

To recall what a Kähler manifold is we start with a given complex manifold M with complex structure J. A Riemannian metric $\langle , \rangle$ on M is called Hermitian, if $\langle JX, JY \rangle = \langle X, Y \rangle$. Then $\omega(X, Y) = \langle JX, Y \rangle$ is a two-form, called the Kähler form of M. The metric is called *Kähler* if $\mathrm{d}\omega = 0$. The geometry of Kähler metrics is a central topic in the differential geometry since its appearance in 1930's. It is related to fundamental branches of mathematics and mathematical physics like algebraic geometry, nonlinear partial differential equations, topology, string theory and others. In recent years the topic is flourishing, thanks to the developments in such areas as extremal Kähler metrics and mirror symmetry. So it is not surprising that several books based on Kähler geometry appeared on these topics.

The book under review is an addition to the existing literature emphasizing on three topics: Calabi conjecture, Kähler hyperbolic spaces and Kodaira embedding theorem. These topics are classical, but after reading the book one should be prepared to enter the recent developments mentioned above. There is certain amount of prerequisites to the reader, which is natural for an advanced topic: basic facts about elliptic partial differential operators, including Sobolev and Hölder spaces, Hodge theory and L^2-index theorem. Some familiarity with differential geometric constructions like vector bundles and characteristic classes is helpful since their discussion is rather brief.

The book contains nine chapters and three appendices. The first six chapters contain general material, necessary to understand the content of the last three. The first chapter sets some notations and conventions for smooth manifolds, vector bundles, Lie derivatives, Riemannian metrics, covariant derivatives, Laplace op-