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# BOOK REVIEW

*Algebraic Models in Geometry*, by Yves Félix, John Oprea and Daniel Tanré, Oxford University Press, Oxford, 2008, xxi + 460pp., ISBN 978-0-19-920652-0.

In his landmark paper [*Infinitesimal Computations in Topology*, Publ. I. H. E. S. **47** (1977) 269-331], D. Sullivan demonstrated the power of differential forms for homotopy theory. Sullivan constructed a version of the de Rahm algebra of forms but with rational coefficients and without any smoothness hypotheses. Remarkably, this construction leads to a complete algebraic model for the homotopy type of a simply connected (or, more generally, a nilpotent) space modulo torsion. Precisely, given any differential graded (DG) algebra $(A, d)$ there is an associated minimal DG algebra $(\wedge V, d)$ (a free algebra with decomposable differential) and a DG algebra map $\varphi\colon \wedge V \to A$ inducing an isomorphism on cohomology. The minimal DG algebra is uniquely determined by $(A, d)$ up to isomorphism. Applying this procedure to Sullivan's de Rahm algebra yields the Sullivan minimal model of a space.

The Sullivan minimal model is a faithful invariant of rational homotopy type: two nilpotent spaces have homotopy equivalent rationalizations if and only if their minimal models are isomorphic. Consequently, all the rational homotopy invariants of a nilpotent space can, in principle, be recovered from its minimal model. While Sullivan's minimal models thus provide an exceptionally powerful tool for homotopy theory, arguably the most striking applications of the theory were originally in geometry. The famous paper of Deligne-Griffiths-Morgan-Sullivan applied Hodge theory and Sullivan's minimal models to prove that the real homotopy type of a Kähler manifold is a formal consequence of its real cohomology. (Sullivan later improved this result in the above mentioned paper, replacing $\mathbb{R}$ with $\mathbb{Q}$.) In another direction, Sullivan models proved decisive in settling an important case of the "closed geodesic problem" for Riemannian manifolds. In the late 1960s, D. Gromoll and W. Meyer proved that a Riemannian manifold $M$ admits infinitely many distinct, periodic geodesics if the free loop space $LM$ has unbounded Betti numbers. Using this result and an explicit Sullivan model for the free loop space,