



BOOK REVIEW

An Introduction to Lie Groups and Lie Algebras, by Alexander Kirillov, Jr. Cambridge University Press, Cambridge, 2008, xi + 222pp., ISBN 978-0-521-88969-8.

The author, of the book under review Alexander Kirillov, Jr. is a faculty member of the Institute for Mathematical Sciences and at the Mathematical Department of Stony Brook University, New York, USA.

The introduction briefly explains the philosophy of the author on how Lie groups and Lie algebras should be taught. It stresses more on the ideas and on the exercises, rather than on the rigorous proofs.

Chapter 2 deals with the basic definitions in Lie groups. The complex Lie group G is introduced as a set with two compatible structures: G is a group and G is a complex analytic manifold. Next in a rather smooth and natural way the important notion of orbits and homogeneous spaces are introduced as well as the definitions of the classical groups.

Chapter 3 deals with the interrelations between Lie groups and Lie algebras \mathfrak{g} . Of course it starts by introducing the main notions of the theory: the exponential map, the Lie group generators and their commutators, the Jacobi identity and the adjoint action of the Lie group G onto its Lie algebra \mathfrak{g} . These are naturally followed by the notions of a subalgebra, ideal and center, by the realization of the Lie algebras in terms of vector fields, and by the formulation of the fundamental theorems in Lie theory. At the end the complex and real forms are introduced.

In Chapter 4 the representation theory of Lie algebras is outlined. The author introduces the notion of representation and irreducible representation, followed by the intertwining operators and Schur's lemma. Next the representation theory for finite groups is briefly formulated, which allows one to explain the character formula and Peter-Weyl theorem. The important example, the representations of the $\mathfrak{sl}(2, \mathbb{C})$ algebra are constructed and then used to analyze the eigenfunctions of the spherical Laplace operator, which are crucial for understanding the spectrum of the hydrogen atom.

Chapter 5 is devoted to the structure theory of the Lie algebras. Naturally it