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BOOK REVIEW

Symplectic Geometry of Integrable Hamiltonian Systems, by Michele Audin, Ana Cannas da Silva and Eugene Lerman, Birkhäuser, Basel, 2003, viii + 225pp, ISBN 3-7643-2167-9

A modern mathematician, or mathematical physicist, could hardly imagine the contemporary mathematics and mathematical physics without differential geometry with all its directions both in pure studies and in applications. The joint work of mathematicians and physicists has resulted in a tremendous advance in many of its branches, for example recall the Riemannian geometry and general relativity during the last century. Another, also very appropriate example is the connection-curvature look on the integrability of partial differential equations in the frame of *fibre bundle theory* and the physical Yang-Mills theory and its later development as general *gauge theory*, where the mathematical *connection* field becomes the carrier of some physical interactions. Physics has always stimulated mathematics to develop new branches, and mathematics has always been very responsive and warm-hearted to the needs of physics. Starting with Newton and Leibniz the greatest mathematicians of all times have contributed considerably to deeply understanding and solving difficult physical problems. One of the most fascinating and impressive examples seems to be the interrelations between *Hamiltonian Mechanics* and *Symplectic Geometry*. Although Hamilton published his view on dynamical equations in 1835, the essence and structure of his approach was adequately understood just in twentieth century, and this would not happen without the work of people like S. Poisson (1781-1840), L. Euler (1707-1783), J. Liouville (1809-1882), J. Lagrange (1736-1813), H. Poincare (1854-1912), and many others among which I would mention S. Lie and E. Cartan with their great achievements in differential geometry and the theory of continuous groups (Lie groups). In this way symplectic geometry was born, i.e. geometry of even dimensional smooth manifolds endowed with a non-degenerate closed two-form ω, $\mathrm{d}\omega = 0$, and the main subject of Hamiltonian mechanics appeared as finding integral trajectories of Hamiltonian vector fields X, i.e. those vector fields which give closed one-forms of the kind $i(X)\omega$. Locally, all such one-forms are exact, i.e., $i(X)\omega = \mathrm{d}H$ and