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BOOK REVIEW

Momentum Maps and Hamiltonian Reduction, by Juan-Pablo Ortega and Tudor S. Ratiu, Birkhäuser, Boston·Basel·Berlin, 2004, xxxiv+497pp, 82,39 €, ISBN: 0-8176-4307-9

A modern mathematician, or mathematical physicist, could hardly imagine the contemporary mathematics and mathematical physics without the concept of symmetry and associated techniques, formulated and used mostly in the modern differential geometry language: this is what is known as Lie group actions on manifolds. From physical point of view this is, of course, understandable, since the real world consists of physical objects, and physical objects show two kind of properties: intrinsic (proper, identifying) and behavioral (kinematical). The identifying properties of an object do not change during its time-evolution, while the behavioral properties describe the admissible changes during evolution. So, this "change-conservation" consistent nature of the physical objects finds its mathematical adequacy in building mathematically realistic dynamical systems, such that their dynamical equations are consistent with corresponding (i.e., physically understandable) invariants. The mathematical tool for finding these invariants is, namely, the concept of "invariance with respect to some Lie group action" on the variables chosen. The local version of this approach, roughly speaking, consists in finding well defined quantities (usually these are some combinations of proper and kinematical characteristics of the system studied) having zero Lie derivatives along the evolution defining objects, which in most cases are given by vector fields on a manifold. As it is well known, having such conservative quantities at hand simplifies the further study of the system, in fact, a corresponding reduction of the independent degrees of freedom is possible. Moreover, in some cases just knowing the conservative quantities of the system, not the detailed kinematics, is of interest. For example, from physical point of view, having true expression for the energy-momentum of the system allows to analyze successfully many aspects of its future development. In symplectic, or Hamiltonian, mechanics the energy-momentum is given by the corresponding "momentum map", which, in view of the physical significance of the energy-momentum as universal and conservative