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BOOK REVIEW

Analytic Hyperbolic Geometry – Mathematical Foundations and Applications, by Abraham A. Ungar, World Scientific, 2005, xvii + 463pp., ISBN 981-256-457-8.

The book under review provides an efficient algebraic formalism for studying the hyperbolic geometry of Bolyai and Lobachevsky, which underlies Einstein special relativity. More precisely, it extracts the properties of Einstein addition of relativistically admissible velocities, varying over the ball $B_c^3 = \{v \in \mathbb{R};\ ||v|| < c\}$ of radius c, equal to the vacuum speed of light, and introduces the notion of a gyrocommutative gyrogroup $(B_c^3, \oplus_E)$. The Einstein addition $\oplus_E$ is not associative and its deviation from associativity is measured by the so called Thomas gyrations. These are rotations from $\mathrm{SO}(3)$, representing the relativistic Thomas precession of the motion of the moon. Moreover, the relativistically admissible velocities $u \in B_c^3$ are associated with Lorentz boosts $L(u)$, i.e., with linear transformations $L(u)$ of the space-time, without a rotation. The deviation of the product of two Lorentz boosts from a Lorentz boost is a space-time rotation, called Thomas precession. As another application of the gyrooperations, the velocity parameter of the Lorentz link between two relativistic space-time events with equal space-time norms is expressed as a cogyrodifference of the velocities of the arguments. Extending the gyroaddition of natural number of copies of one and a same gyrovector to a scalar multiplication by real numbers, the author turns any abstract gyrocommutative gyrogroup $(G, \oplus)$ into a gyrovector space $(G, \oplus, \otimes)$.

Classically, the vector algebra provides the necessary algebraic formalism for analytic description of the Euclidean geometry. In a similar vein, Ungar's gyrovector spaces constitute the algebraic background for solving qualitative and quantitative problems from the hyperbolic geometry. In physics, the Newtonian mechanics is modelled on the Euclidean geometry, so that the usual vector addition and its associated analytic expression account for the addition of Newtonian velocities. Respectively, Einstein's relativistic space is assumed to be hyperbolic and the gyroaddition of the gyrovectors extract the properties of Einstein's addition of relativistically admissible velocities.