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BOOK REVIEW

Cycle Spaces of Flag Domains – A Complex Geometric Viewpoint, by Gregor Fels, Alan Huckleberry and Joseph A. Wolf, Birkhäuser, Boston · Basel · Berlin, 2006, xx + 329 pp., ISBN 0-8176-4391-5.

The research monograph under review constructs cycle spaces of complex flag domains and studies their complex-analytic properties. The results are expected to through a bridge between the representation theory of semisimple Lie groups and the variations of Hodge structure of compact complex manifolds. Throughout the book, G is a complex simple Lie group and Q is a parabolic subgroup of G. Then the projective algebraic manifold $Z = G/Q$ is called a flag manifold. If G_0 is a real form of G then any open G_0-orbit $D = G_0(z_0)$ of the base point $z_0 \in Z$ is referred to as a flag domain. For any maximal compact subgroup K_0 of G_0, there is a unique K_0-orbit C_0 of D, which is a complex submanifold of Z and C_0 is known as the base cycle of D. Let us denote by $\mathcal{C}_q(X)$ the space of the q-dimensional cycles of a complex space X. Then $\mathcal{C}_q(Z)$ is a locally finite-dimensional complex space with compact projective irreducible components. In the case of $q := \dim_{\mathbb{C}} C_0$ consider the connected component $\mathcal{C}$ of $\mathcal{C}_q(D)$, containing C_0, and its subspace $\mathcal{M}_D$, defined as the connected component of $G(C_0) \cap \mathcal{C}_q(D)$ through C_0. Let K be the complexification of K_0 and $\Omega = G/K$ be the corresponding affine homogeneous space. Except for some noncompact Hermitian symmetric D, the cycle space $\mathcal{M}_D$ turns to be isomorphic to the maximal, G_0-invariant, Kobayashi-hyperbolic, Stein domain in Ω through the base point $\check{o}_\Omega$, corresponding to C_0. If D is a bounded symmetric domain then $\mathcal{M}_D$ coincides with D, its complex conjugate $\overline{D}$ or their Cartesian product $D \times \overline{D}$. The monograph establishes that $\mathcal{C}$ is smooth and provides a complete representation-theoretic description of the Zariski tangent space $T_{[C_0]}\mathcal{C}$, viewed as a K-module. For an arbitrary G_0-orbit γ on Z, the authors define a cycle space $\mathcal{C}(\gamma)$, realized as an open subset of a G-orbit on $\mathcal{C}_q(Z)$. In particular, $\mathcal{C}(D) = \mathcal{M}_D$ for the open G_0-orbit D on Z. It turns out that except for some well understood hermitian symmetric cases, $\mathcal{C}(\gamma)$ are isomorphic to some explicitly constructed universal domains $\mathcal{U}$, introduced by Akhiezer and Gindikin