Homology, Homotopy and Applications, vol. 8(2), 2006, pp.105–114

THE ETA INVARIANT AND THE "TWISTED" CONNECTIVE K-THEORY OF THE CLASSIFYING SPACE FOR CYCLIC 2-GROUPS

EGIDIO BARRERA-YANEZ

(*communicated by Jonathan Rosenberg*)

Abstract

Let $\ell = 2^\nu \geqslant 2$. We use the eta invariant to study the "twisted" connective real K-theory groups $ko_m(B\mathbb{Z}_\ell, \xi_1)$ of the classifying space $B\mathbb{Z}_\ell$ for the cyclic group $\mathbb{Z}_\ell$.

1. Introduction

Atiyah [**1**] expressed the complex K-theory of the classifying space of $\mathbb{Z}_\ell$ in terms of the complex representation ring of $\mathbb{Z}_\ell$. Looking for a "geometric" construction of Elliptic homology, Kreck and Stolz [**11**] gave a geometric characterization of connective real K-theory. Stolz used this characterization to study when a simply connected manifold admits a metric with positive scalar curvature, see [**12**] for details. Botvinnik and Gilkey [**6**] and Botvinnik, Gilkey, and Stolz [**7**] studied when a manifold with non-trivial fundamental group admits a metric with positive scalar curvature. They defined a "twisted" geometric version of connective real K-theory. In this paper we will express the "twisted" connective real K-theory of the classifying space $\mathbb{Z}_\ell$ in terms of the complex representation ring of $\mathbb{Z}_\ell$. Instead of using topological methods as Atiyah did, we shall use analytical methods; our fundamental tool is the eta invariant.

Let $\mathbb{Z}_\ell := \{\lambda \in \mathbb{C} : \lambda^\ell = 1\}$ be the cyclic group of order $\ell = 2^\nu \geqslant 2$. Let $\rho_s(\lambda) = \lambda^s$ define an irreducible linear representation of $\mathbb{Z}_\ell$. The ρ_s parametrize the irreducible representations of $\mathbb{Z}_\ell$. Let ξ_1 be the underlying real 2 plane bundle of the complex line bundle defined by the representation ρ_1. Let $D(\xi_1)$, $S(\xi_1)$ be the disk bundle and respectively sphere bundle with respect to some fiber metric on ξ_1. Let $T(\xi_1) = D(\xi_1)/S(\xi_1)$ be the Thom space associated with ξ_1. We use the Thom-Pontryagin construction to define the twisted equivariant spin bordism groups and twisted connective real K-theory groups by:

$$MSpin_m(B\mathbb{Z}_\ell, \xi_1) := \widetilde{MSpin}_{m+2}(T(\xi_1))$$
$$ko_m(B\mathbb{Z}_\ell, \xi_1) := \widetilde{ko}_{m+2}(T(\xi_1)).$$

---

†The author passed away on August 19, 2004.

Received January 12, 2004; published on August 9, 2006.
2000 Mathematics Subject Classification: 55N15, 58G12.
Key words and phrases: connective K-theory, eta invariant.