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ADDENDUM TO "INTRODUCTION TO A-INFINITY ALGEBRAS AND MODULES"

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(*communicated by Lionel Schwartz*)

I thank J. Huebschmann for his detailed comments on the article [**15**]. With the help of his letter [**14**], I have compiled the following list of additions and corrections to be made in the respective sections of [**15**]:

1.2 History. An A-infinity structure may be described as a system of higher homotopies together with suitable coherence conditions. A basic observation, which has implicitly been exploited for long, is that A-infinity structures behave much better with respect to homotopy than strict (for example differential graded algebra) structures. Higher homotopies occurred in mathematics before A-infinity structures had been explicitly recognized, though. The system of $\cup_i$-products introduced by Steenrod [**19**] is an early example of higher homotopies. The $\cup_i$-products measure the failure from commutativity of the Alexander-Whitney map in a coherent fashion and prompted the development of s(trongly)h(omotopy)c(ommutative) structures as well as that of Steenrod operations. Massey products [**16**] may be seen as invariants of certain A-infinity structures. Homological perturbation theory (HPT) has nowadays become a standard tool to construct and handle A-infinity structures. The basic HPT-notion, that of contraction, was introduced in Section 12 of [**3**]. In that paper, Eilenberg and Mac Lane showed that the comparison between the reduced bar and W-constructions is a reduction, and they conjectured that it is a contraction. Using the perturbation lemma, in his Heidelberg diploma thesis (Diplomarbeit) supervised by J. Huebschmann, Wong has indeed verified this conjecture [**20**]. A geometric comparison between the bar and W-constructions has recently been obtained by Berger and Huebschmann in [**1**]. The notion of "recursive structure of triangular complexes" in Section 5 [**4**] is also an example of what was later identified as a perturbation. The "perturbation lemma" is lurking behind the formulas in Chapter II of Section 1 of [**18**] and seems to have first been made explicit by M. Barrat (unpublished). The first instance known to us where it appeared in print is [**2**]. A homological algebra and higher homotopies tradition was as well created by Berikashvili and his students in Georgia (at the time part of the USSR). More precise comments about the historical development until the mid eighties may be

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