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ON THE DISTRIBUTION OF THE EIGENVALUES OF A CLASS OF INDEFINITE EIGENVALUE PROBLEMS

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Abstract. We prove detailed asymptotic estimates for the distribution of the eigenvalues of linear boundary eigenvalue problems of arbitrary order n with indefinite weight function generalizing well known results for the case $n = 2$.

1. Introduction. We consider eigenvalue problems of the form

$$\ell(y) = y^{(n)} + \sum_{\nu=2}^{n} f_\nu(x) y^{(n-\nu)} = \lambda r(x) y, \quad x \in [0,1] \tag{1.1}$$

$$U_\nu(y) = U_{\nu 0}(y) + U_{\nu 1}(y) = 0, \quad \nu = 1, \cdots, n, \tag{1.2}$$

where $r : [0,1] \to \mathbb{R}\backslash\{0\}$ is a step function; $f_\nu \in L[0,1]$, $2 \le \nu \le n$, and where the boundary conditions are normalized; the latter means that

$$\begin{aligned} U_{\nu 0}(y) &= \alpha_\nu y^{(k_\nu)}(0) + \sum_{\mu=0}^{k_\nu - 1} \alpha_{\nu\mu} y^{(\mu)}(0), \\ U_{\nu 1}(y) &= \beta_\nu y^{(k_\nu)}(1) + \sum_{\mu=0}^{k_\nu - 1} \beta_{\nu\mu} y^{(\mu)}(1), \end{aligned} \tag{1.3}$$

$$|\alpha_\nu| + |\beta_\nu| > 0 \quad \text{for} \quad \nu = 1, \cdots, n,$$

$$n - 1 \ge k_1 \ge k_2 \ge \cdots \ge k_n \ge 0 \quad \text{with} \quad k_\nu > k_{\nu+2} \ \text{ for } \ \nu = 1, \cdots, n-2.$$

A central role in our paper is played by the assumption that the boundary conditions (1.2) are regular; cf. Definition 2 and Definition 7, where the definition of Birkhoff-regularity for definite problems (Naimark [12, p. 56]) is generalized in a natural

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