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AN EXTENSION OF LERAY–SCHAUDER DEGREE AND APPLICATIONS TO NONLINEAR WAVE EQUATIONS

JUHA BERKOVITS AND VESA MUSTONEN
Department of Mathematics, University of Oulu, SF-90570 Oulu, Finland

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Abstract. In this paper we introduce a construction of the classical topological degree for a class of mappings of monotone type motivated by the periodic Dirichlet problem for the semilinear wave equation

$$\begin{cases} u_{tt} - u_{xx} + g(t,x,u) = h(t,x), \ (t,x) \in \mathbf{R} \times (0,\pi) \\ u(t,0) = u(t,\pi) = 0 \text{ for all } t \in \mathbf{R} \\ u \text{ is } 2\pi\text{-periodic in } t. \end{cases} \tag{SWE}$$

Using homotopy arguments of the degree theory we derive existence theorems for the abstract setting which can be applied to the problem (SWE).

1. Introduction. Let H be a real separable Hilbert space and L a closed densely defined linear operator: $H \supset D(L) \to H$ having the property $\operatorname{Im} L = (\operatorname{Ker} L)^{\perp}$. Hence $\operatorname{Im} L$ is closed and $\operatorname{Ker} L = \operatorname{Ker} L^*$. Let L_0 stand for the restriction of L into $\operatorname{Im} L$ and assume that $L_0^{-1} : \operatorname{Im} L \to \operatorname{Im} L$ is compact. We shall study the existence of solutions of the nonlinear equation

$$Lu + N(u) = h, \tag{SE}$$

where the nonlinearity $N : H \to H$ is assumed to belong to some class of mappings of monotone type. The above properties of L are modelled by the differential operator $\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}$ appearing in the semilinear wave equation (SWE).

Our study is based on the topological degree theory which we shall construct for a class of mappings $F : H \to H$ of the type

$$F = Qg + Pf,$$

where Q and P are the orthogonal projections to $\operatorname{Im} L$ and $\operatorname{Ker} L$, respectively, $g = I + C$ is some Leray-Schauder map with C compact and f is a mapping of class (S_+). Existence theorems for the equation

$$F(u) = y \tag{E}$$

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