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MATHEMATICAL STUDY OF A NONLINEAR TRANSPORT-DIFFUSION PROBLEM RELATED TO MUSCLE CONTRACTION

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Abstract. We analyse a transport-diffusion initial-boundary value problem where the coefficient of the transport term is a time function depending on the solution in a nonlinear and nonlocal way. Using fixed-point techniques, we prove an uniqueness and local existence result. The problem is related to a new model describing muscle contraction in the framework of sliding filaments theory, including the slipping effects in the cross-bridge dynamics.

1. Introduction. In this paper we study from a mathematical point of view the following nonlinear and nonlocal initial-boundary value problem.

Find a function $u : [-1,1] \times [0,T] \to \mathbb{R}$ satisfying

$$u_t - \big(\varepsilon(x)u_x\big)_x + \dot{z}(t)u_x = \varphi\big(x,t,z(t),u\big) \quad \text{a.e. in } [-1,1] \times [0,T], \tag{1.1}$$

$$u(-1,t) = u(1,t) = 0 \qquad \text{for } t \in [0,T], \tag{1.2}$$

$$u(x,0) = u_0(x) \qquad \text{for } x \in [-1,1], \tag{1.3}$$

where the dot denotes the time derivative and the function $z : [0,T] \to \mathbb{R}$ is related to u by the nonlocal equation

$$z(t) = L\Big(\int_{-1}^{1} w(x)u(x,t)\,dx\Big) \quad \text{for } t \in [0,T]. \tag{1.4}$$

Here $T > 0$, $\varepsilon : [-1,1] \to \mathbb{R}$ is a strictly positive function, φ is Lipschitz continuous with respect to $z(t)$ and u, L is a smooth (in a sense to be made precise later)

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