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A WEIGHTED SEMILINEAR ELLIPTIC EQUATION INVOLVING CRITICAL SOBOLEV EXPONENTS

LIVIU I. NICOLAESCU†
Faculty of Mathematics, University of Iaşi, 6600 Iasi, Romania

(Submitted by: V. Barbu)

Abstract. In this paper we prove the existence of a positive radial solution of the problem

$$-\Delta u = r^{\sigma}|u|^{p-1}u + \lambda r^{\alpha}u, \quad \text{in } B_R \subset \mathbf{R}^N \ (r = |x|)$$

for λ in a suitable (and almost optimal) range. Here $N \geq 3$, $\alpha, \sigma \geq -2$ and $p = (N+2+2\sigma)/(N-2)$ corresponds to the critical Sobolev exponent $p+1 = (2N+2\sigma)/(N-2)$. Our result extends the previous one due to Brézis and Nirenberg when $\sigma = \alpha = 0$.

0. Introduction. In a previous paper [8] we considered the problem

$$\begin{cases} -\dfrac{1}{r^{\gamma}}(r^{\gamma}u')' = r^{\sigma}|u|^{q-1}u \quad \text{in } (0,1) \\ u(1) = 0, \quad \displaystyle\int_0^1 r^{\gamma}|u'|^2\,dr < \infty \\ u > 0. \end{cases} \tag{0.1}$$

We recall some of the results we obtained there.

"If $\gamma > 1$ then the problem has exactly one weak solution for $1 < q < \frac{\gamma+3+2\sigma}{\gamma-1}$ and no weak solution for $q > (\gamma+3+2\sigma)/(\gamma-1)$."

In this paper we shall deal exactly with the critical case, namely, $q = p = (\gamma + 3 + 2\sigma)/(\gamma - 1)$. Instead of (0.1) we consider the more general problem

$$\begin{cases} -\dfrac{1}{r^{\gamma}}(r^{\gamma}u')' = r^{\sigma}|u|^{p-1}u + \lambda r^{\alpha}u \quad \text{in } (0,\,1) \\ u(1) = 0, \quad \displaystyle\int_0^1 r^{\gamma}|u'|^2\,dr < \infty \\ u > 0 \end{cases} \tag{0.2}$$

where $\gamma > 1$, $\sigma, \alpha > -2$.

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†Current address: Department of Mathematics, Michigan State University, East Lansing, MI 48824.
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