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BOUNDS FOR SOLUTIONS OF INTEGRAL EQUATIONS OF SECOND TYPE

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Abstract. Bounds are given for the solutions of the Volterra and Fredholm integral equations of the second type. These bounds are also valid for nonlinear equations. The methods of the paper can also be used for Volterra equations of the first kind.

1. In none of the standard works on integral equations does any formula govering upper and lower bounds on the solutions $\phi(x)$ of the integral equations

$$\phi(x) = f(x) + \int_a^x F(x, y, \phi(y))\, dy \tag{1.1a}$$

and

$$\phi(x) = f(x) + \int_a^b F(x, y, \phi(y))\, dy \tag{1.1b}$$

appear to be given, even in the linear case

$$f(x, y, \phi(x)) = K(x, y)\phi(y). \tag{1.1c}$$

Clearly such bounds can be determined if it is possible to make an estimate of the modulus of the integral which is independent of ϕ. It will be shown that this will be possible when $F(x, y, \phi(y))$ obeys a relation of the form

$$|F(x, y, \phi(y))| \leq m(x)n(y)|\phi(y)| \tag{1.2a}$$

or a generalized Lipschitz condition of the form

$$|F(x, y, z_1) - F(x, y, z_2)| \leq p(x)q(y)|z_1 - z_2|; \tag{1.2b}$$

m, n, p, q are all positive and can be constants. In the linear case, it is clear that $m = p$ and $n = q$.

2. Consider first the Volterra-type equation. It follows immediately from equation (1.1a) that

$$|\phi(x) - f(x)| = \left| \int_a^x F(x, y, \phi(y))\, dy \right| \tag{2.1}$$

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