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SOME COMPARISON PRINCIPLES FOR WEAKLY COUPLED SYSTEMS OF ELLIPTIC EQUATIONS

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Abstract. Consider a system of weakly coupled elliptic partial differential equations where each equation in the system involves the same uniformly elliptic operator. We introduce several transformations which change the system into a system for which the classical extremum principle holds. This leads to pointwise comparison results of the component functions and, under additional assumptions, to positive solutions of the Dirichlet problem for the system.

1. Introduction. In a recent paper [9], the method of decoupling a weakly coupled system of two elliptic partial differential equations of second order was utilized to determine the algebraic sign of the solution functions for the Dirichlet problem for the system. This work expanded and completed a remark of de Figueiredo and Mitidieri [3] which occurred in their study of the existence of positive solutions to a semilinear elliptic system that represents the steady state situation in a reaction diffusion problem of interest in biology (see [4], [5], [7], and [8]).

The results in [3] and [9] are consequences of the maximum principle for the decoupled equations. As a system, one might appeal to the classical maximum principle of Protter and Weinberger [6, p. 192] for elliptic systems of the form

$$\sum_{i,j=1}^{n} a_{ij}^{(k)} \frac{\partial^2 u_k}{\partial x_i \partial x_j} + \sum_{i=1}^{n} b_i^{(k)} \frac{\partial u_k}{\partial x_i} + \sum_{\ell=1}^{m} h_{k\ell} u_\ell \geq 0, \quad k = 1, 2, \ldots, m,$$

where the off-diagonal elements $h_{k\ell}$, $k \neq \ell$, of the coupling matrix H are assumed to be nonnegative and the row sums $\sum_{\ell=1}^{m} h_{k\ell}$, $k = 1, 2, \ldots, m$, are nonpositive.

The system of two equations considered in [3] involved the same elliptic operator in each equation and a constant coupling matrix (see also [10]). While the condition that the elliptic operators be the same in each equation consititutes a restriction of the classical maximum principle for weakly coupled elliptic systems [6], the allowance for the off-diagonal elements of the coupling matrix to be of opposite sign constitutes alternative sufficient conditions under which an extremum principle can

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